NON-LINEAR ANALYSIS OF SMART COMPOSITE STRUCTURAL SYSTEMS WITH EMBEDDED SENSORS

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Contract: DAAE07-01-P-L023

September 6, 2003

1. OBJECTIVES

The objective of the research is to develop efficient computational tools for non-linear analysis of smart composites structural systems with embedded smart (e.g., piezoelectric or magnetostrictive particle) layers. The main thrust of the contract is to develop analytical as well as computational (i.e., finite element analysis) procedures for the stress and vibration response of laminated composite plate structures with smart-material layers. The first and second phases of the research was concerned with (1) the theoretical formulation and analytical solutions for vibration suppression of laminated plates and (2) the linear finite element analysis of laminated composite plates with smart-material layers, respectively. This report summarizes the work carried out in the third and last phase of the contract; in particular, it summarizes the linear transient analysis using the FEM and non-linear finite element formulation based on the third-order shear deformation plate theory.

2. INTRODUCTION

The study of smart materials and structures has received considerable attentions in recent years. The advantage of incorporating these special types of materials into the structure is that the sensing and actuating mechanism becomes part of the structure so that one can monitor the structural integrity/health of the structure. There are a number of materials that have the capability to be used as a sensor or an actuator or both. Piezoelectric materials, magnetostrictive materials, electrostrictive materials, shape memory alloys, and electrorheological fluids provide examples of such materials. Among these, piezoelectric and magnetostrictive materials have the capability to serve as both sensors and actuators. Piezoelectricity [1] is a phenomenon in which some materials develop polarization upon application of strains. Examples of piezoelectric materials are Rochelle salt, quartz, and Lead Zirconate Titanate or PZT (Pb (Zr,Ti) O_3). Piezoelectric materials exhibit a linear relationship between the electric field and strains for low field values (up to 100 V/mm); and it exhibits nonlinear behavior for large fields, and the material exhibits hysteresis [2]. Furthermore, piezoelectric materials show dielectric aging and hence lack reproducibility of strains, i.e., a drift from zero state of strain is observed under cyclic electric field conditions. Terfenol-D, a

Report Documentation Page			*Form Approved OMB No. 0704-0188*
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.			
1. REPORT DATE **06 SEP 2003**	2. REPORT TYPE **Technical Report**		3. DATES COVERED **04-01-2003 to 16-08-2003**
4. TITLE AND SUBTITLE **NON-LINEAR ANALYSIS OF SMART COMPOSITE STRUCTURAL SYSTEMS WITH EMBEDDED SENSORS**			5a. CONTRACT NUMBER **DAAE07-01-P-L023** 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) **J Reddy; F Rostam-Abadi**			5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Department of Mechanical Engineering,Texas A&M University ,College Station,TX,77840**			8. PERFORMING ORGANIZATION REPORT NUMBER **; #14015**
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) **U.S. Army TARDEC, 6501 East Eleven Mile Rd, Warren, Mi, 48397-5000**			10. SPONSOR/MONITOR’S ACRONYM(S) **TARDEC** 11. SPONSOR/MONITOR’S REPORT NUMBER(S) **#14015**
12. DISTRIBUTION/AVAILABILITY STATEMENT **Approved for public release; distribution unlimited**			
13. SUPPLEMENTARY NOTES			

14. ABSTRACT

The objective of the research is to develop efficient computational tools for non-linear analysis of smart composites structural systems with embedded smart (e.g., piezoelectric or magnetostrictive particle) layers. The main thrust of the contract is to develop analytical as well as computational (i.e., finite element analysis) procedures for the stress and vibration response of laminated composite plate structures with smart-material layers. The first and second phases of the research was concerned with (1) the theoretical formulation and analytical solutions for vibration suppression of laminated plates and (2) the linear finite element analysis of laminated composite plates with smart-material layers, respectively. This report summarizes the work carried out in the third and last phase of the contract; in particular, it summarizes the linear transient analysis using the FEM and non-linear finite element formulation based on the third-order shear deformation plate theory.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Public Release**	**30**	

magnetostrictive material [3], has the characteristics of being able to produce strains up to 2500μm and energy density as high as 25000 J/m^3 in response to a magnetic field.

Vibration and shape control of flexible structures is achieved with the help of actuators and a feedback control law. Many modern techniques have been developed in recent years to meet the challenge of designing controllers that suit the function under some required conditions. There have been a number of studies on vibration control of flexible structures using smart materials. Anjanappa and Bi [4,5] investigated the feasibility of using embedded magnetostrictive mini actuators for smart structure applications, such as vibration suppression of beams. A self-sensing magnetostrictive actuator design based on a linear model of magnetostrictive transduction for Terfenol-D was developed and analyzed by Pratt and Flatau [6]. Eda et al [7] and Krishna Murty et al [8,9] proposed magnetostrictive actuators that take advantage of the ease with which the actuators can be embedded and remote excitation capability of magnetostrictive particles as actuators for smart structures. Reddy and Barbosa [10] presented a general formulation and analytical solution for simply supported boundary conditions of laminated composite beams with embedded magnetostrictive layers. All these studies were based on free vibration analysis of the structures with velocity feedback control.

In the present study, control of the transient response of laminated composite plates with integrated smart material layers, used as sensors and actuators, is studied using a unified plate theory that includes the classical, first-order and third-order plate theories as special cases. A simple negative velocity feedback control is used to actively control the dynamic response of the structure through a close-loop control. A displacement finite element model of the equations of motion governing the unified theory is developed. The effects of material properties, lamination scheme, and placement of the smart material layer on deflection suppression are investigated.

3. THEORETICAL FORMULATION

3.1 Introduction

The simplest equivalent single-layer theories are the classical laminate plate theory (CLPT) and the first-order shear deformation theory (FSDT). These theories describe the kinematics of most laminated plates adequately. The third–order shear deformation theory (TSDT) represents the plate kinematics better and yields better inter-laminar stress distributions. Quadratic variations of the transverse shear strains and stresses through the layer avoid the need for shear correction coefficients as required in the first-order theory.

3.2 Displacement field

The displacement field for the third-order shear deformation theory (TSDT) can be expressed in the form (see Reddy [12])

$$u(x,y,z,t) = u_0(x,y,t) + z\phi_x(x,y,t) - c_1 z^3(\phi_x + \frac{\partial w_0}{\partial x}) \tag{1}$$

$$v(x,y,z,t) = v_0(x,y,t) + z\phi_y(x,y,t) - c_1 z^3(\phi_y + \frac{\partial w_0}{\partial y}) \tag{2}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{3}$$

where (u_0, v_0, w_0) and (ϕ_x, ϕ_y) have the same physical meaning as in the first–order theory; they denote the displacements and rotations of a transverse normal on the plane $z = 0$,

respectively. Then the displacement field of FSDT is obtained by setting $c_1 = 0$, and for the Reddy third-order theory it is equal to $c_1 = 4/3h^2$.

3.3 Equations of motion

The equations of motion of the third-order shear deformation theory are derived using the dynamic version of the principle of virtual displacements. They are (see Reddy [12])

$$\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\frac{\partial^2 u_0}{\partial t^2}+J_1\frac{\partial^2 \phi_x}{\partial t^2}-c_1 I_3\frac{\partial^2}{\partial t^2}\left(\frac{\partial w_0}{\partial x}\right) \tag{4}$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_{yy}}{\partial y}=I_0\frac{\partial^2 v_0}{\partial t^2}+J_1\frac{\partial^2 \phi_y}{\partial t^2}-c_1 I_3\frac{\partial^2}{\partial t^2}\left(\frac{\partial w_0}{\partial y}\right) \tag{5}$$

$$\begin{gathered}\frac{\partial \overline{Q}_x}{\partial x}+\frac{\partial \overline{Q}_y}{\partial y}+c_1\left(\frac{\partial^2 P_{xx}}{\partial x^2}+2\frac{\partial^2 P_{xy}}{\partial x\partial y}+\frac{\partial^2 P_{yy}}{\partial y^2}\right)+q= \\ I_0\frac{\partial^2 w_0}{\partial t^2}-c_1^2 I_6\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial y^2}\right)+c_1\left[I_3\frac{\partial^2}{\partial t^2}\left(\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}\right)+J_4\frac{\partial^2}{\partial t^2}\left(\frac{\partial \phi_x}{\partial x}+\frac{\partial \phi_y}{\partial y}\right)\right]\end{gathered} \tag{6}$$

$$\frac{\partial \overline{M}_{xx}}{\partial x}+\frac{\partial \overline{M}_{xy}}{\partial y}-\overline{Q}_x=\frac{\partial^2}{\partial t^2}\left(J_1 u_0+K_2\phi_x-c_1 J_4\frac{\partial w_0}{\partial x}\right) \tag{7}$$

$$\frac{\partial \overline{M}_{xy}}{\partial x}+\frac{\partial \overline{M}_{yy}}{\partial y}-\overline{Q}_y=\frac{\partial^2}{\partial t^2}\left(J_1 v_0+K_2\phi_y-c_1 J_4\frac{\partial w_0}{\partial y}\right) \tag{8}$$

where

$$\overline{M}_{\alpha\beta}=M_{\alpha\beta}-c_1 P_{\alpha\beta}\,,\ \overline{Q}_\alpha=Q_\alpha-3c_1 R_\alpha$$

$$I_i=\sum_{k=1}^{N}\int_k^{k+1}\rho^{(k)}(z)^i\,dz\quad (i=0,1,2,\dots,6)\,,\ J_i=I_i-c_1\,I_{i+2}$$

$$K_2=I_2-2c_1 I_4+c_1^2 I_6\,,\ c_1=\frac{4}{3h^2}\,,\ c_2=3c_1 \tag{9}$$

and (N_{xx}, N_{yy}, N_{xy}) denote the total in-plane force resultants, (M_{xx}, M_{yy}, M_{xy}) the moment resultants, and (P_{xx}, P_{yy}, P_{xy}) and (R_x, R_y) denote the higher-order stress resultants.

$$\begin{Bmatrix}N_{xx}\\N_{yy}\\N_{xy}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{xx}\\\sigma_{yy}\\\sigma_{xy}\end{Bmatrix}dz\,,\ \begin{Bmatrix}M_{xx}\\M_{yy}\\M_{xy}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{xx}\\\sigma_{yy}\\\sigma_{xy}\end{Bmatrix}z\,dz\,,\ \begin{Bmatrix}P_{xx}\\P_{yy}\\P_{xy}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{xx}\\\sigma_{yy}\\\sigma_{xy}\end{Bmatrix}z^3dz$$

$$\begin{Bmatrix}R_y\\R_x\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{xz}\\\sigma_{yz}\end{Bmatrix}z^2dz\,,\ \begin{Bmatrix}Q_y\\Q_x\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{xz}\\\sigma_{yz}\end{Bmatrix}dz \tag{10}$$

The force and moment resultants are related to the strains by

$$\begin{Bmatrix}\{N\}\\\{M\}\\\{P\}\end{Bmatrix}=\begin{bmatrix}[A] & [B] & [E]\\ [B] & [D] & [F]\\ [E] & [F] & [H]\end{bmatrix}\begin{Bmatrix}\{\varepsilon^{(0)}\}\\\{\varepsilon^{(1)}\}\\\{\varepsilon^{(3)}\}\end{Bmatrix}-\begin{Bmatrix}\{N^{M}\}\\\{M^{M}\}\\\{P^{M}\}\end{Bmatrix},\quad \begin{Bmatrix}\{Q\}\\\{R\}\end{Bmatrix}=\begin{bmatrix}[A] & [D]\\ [D] & [F]\end{bmatrix}\begin{Bmatrix}\{\gamma^{(0)}\}\\\{\gamma^{(2)}\}\end{Bmatrix} \tag{11}$$

$$\left(A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij}\right)=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\overline{Q}_{ij}^{(k)}(1,z,z^2,z^3,z^4,z^6)dz \tag{12}$$

where the stiffnesses A_{ij}, D_{ij}, and F_{ij} are defined for i, j = 1, 2, 6 as well as i, j = 4, 5. The stiffnesses B_{ij}, E_{ij}, and H_{ij} are defined only for i, j = 1, 2, 6. The coefficients of A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, and H_{ij} are given in terms of the layer stiffnesses $\overline{Q}_{ij}$ and layer coordinates z_{k+1} and z_k.

The stress resultants associated with the magnetostrictive materials, $\{N^M\}$, $\{M^M\}$, and $\{P^M\}$ are defined by

$$\begin{Bmatrix}N_{xx}^{M}\\N_{yy}^{M}\\N_{xy}^{M}\end{Bmatrix}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}\overline{e}_{31}\\\overline{e}_{32}\\\overline{e}_{36}\end{Bmatrix}^{(k)}H_z\ dz,\quad \begin{Bmatrix}M_{xx}^{M}\\M_{yy}^{M}\\M_{xy}^{M}\end{Bmatrix}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}\overline{e}_{31}\\\overline{e}_{32}\\\overline{e}_{36}\end{Bmatrix}^{(k)}H_z\ z\ dz$$

$$\begin{Bmatrix}P_{xx}^{M}\\P_{yy}^{M}\\P_{xy}^{M}\end{Bmatrix}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}\overline{e}_{31}\\\overline{e}_{32}\\\overline{e}_{36}\end{Bmatrix}^{(k)}H_z\ z^3\ dz \tag{13}$$

where $\overline{e}_{ij}$ is the transformed moduli of the actuating/sensing material, which in the present study is taken to be a magnetostrictive material , and H_z is the electric field intensity which should be excluded in the constitutive relations for the structural part of the composite structures.

3.4 Velocity feedback control

Considering velocity proportional closed-loop feedback control, the magnetic field intensity H can be expressed in terms of coil constant k_c and coil current $I(x,y,t)$ as

$$H(x,y,t)=k_c\ I(x,y,t) \tag{14}$$

and

$$k_c = \frac{n_c}{\sqrt{b_c^2 + 4r_c^2}} \;, \qquad I(x,y,t) = c(t)\frac{\partial w_0}{\partial t} \tag{15}$$

where b_c is the coil width , r_c is coil radius, n_c is number of turns in the coil, and $c(t)$ is the control gain which is assumed to be a constant in this study.

4. FINITE ELEMENT MODEL (LINEAR)
4.2 Virtual work statements

The weak forms of the equilibrium equations are

$$0 = \int_{\Omega^e} \left\{ \frac{\partial \delta u_0}{\partial x} N_{xx} + \frac{\partial \delta u_0}{\partial y} N_{xy} + \delta u_0 \left[I_0 \frac{\partial^2 u_0}{\partial t^2} + J_1 \frac{\partial^2 \phi_x}{\partial t^2} - c_1 I_3 \frac{\partial^2}{\partial t^2}\left(\frac{\partial w_0}{\partial x}\right)\right]\right\} dxdy$$
$$- \oint_\Gamma \{\delta u_0 (N_{xx} n_x + N_{xy} n_y)\} ds \tag{16}$$

$$0 = \int_{\Omega^e} \left\{ \frac{\partial \delta v_0}{\partial x} N_{xy} + \frac{\partial \delta v_0}{\partial y} N_{yy} + \delta v_0 \left[I_0 \frac{\partial^2 v_0}{\partial t^2} + J_1 \frac{\partial^2 \phi_y}{\partial t^2} - c_1 I_3 \frac{\partial^2}{\partial t^2}\left(\frac{\partial w_0}{\partial y}\right)\right]\right\} dxdy$$
$$- \oint_\Gamma \{\delta v_0 (N_{xy} n_x + N_{yy} n_y)\} ds \tag{17}$$

$$0 = \int_{\Omega^e} \left\{ \frac{\partial \delta w_0}{\partial x} \overline{Q}_x + \frac{\partial \delta w_0}{\partial y} \overline{Q}_y - c_1 \left(\frac{\partial^2 \delta w_0}{\partial x^2} P_{xx} + 2\frac{\partial^2 \delta w_0}{\partial x \partial y} P_{xy} + \frac{\partial^2 \delta w_0}{\partial y^2} p_{yy} \right) - \delta w_0 q \right.$$
$$+ \left(\delta w_0 I_0 \frac{\partial^2 w_0}{\partial t^2} + c_1^2 I_6 \left(\frac{\partial \delta w_0}{\partial x} \frac{\partial^3 w_0}{\partial x \partial t^2} + \frac{\partial \delta w_0}{\partial y} \frac{\partial^3 w_0}{\partial y \partial t^2} \right) \right.$$
$$\left.\left. - c_1 \left[I_3 \left(\frac{\partial \delta w_0}{\partial x} \frac{\partial^2 u_0}{\partial t^2} + \frac{\partial \delta w_0}{\partial y} \frac{\partial^2 v_0}{\partial t^2} \right) + J_4 \left(\frac{\partial \delta w_0}{\partial x} \frac{\partial^2 \phi_x}{\partial t^2} + \frac{\partial \delta w_0}{\partial y} \frac{\partial^2 \phi_y}{\partial t^2} \right) \right] \right) \right\} dxdy$$
$$- \oint_\Gamma \delta w_o \overline{V}_n ds - \oint_\Gamma \frac{\partial \delta w_o}{\partial n} P_{nn} ds \tag{18}$$

$$0 = \int_{\Omega^e} \left\{ \frac{\partial \delta \phi_x}{\partial x} \overline{M}_{xx} + \frac{\partial \delta \phi_x}{\partial y} \overline{M}_{xy} + \delta \phi_x \overline{Q}_x + \delta \phi_x \left[\frac{\partial^2}{\partial t^2} \left(J_1 u_0 + K_2 \phi_x - c_1 J_4 \frac{\partial w_0}{\partial x} \right) \right] \right\} dxdy$$
$$- \oint_\Gamma \{\delta \phi_x (\overline{M}_{xx} n_x + \overline{M}_{xy} n_y)\} ds \tag{19}$$

$$0 = \int_{\Omega^e} \left\{ \frac{\partial \delta \phi_y}{\partial x} \overline{M}_{xy} + \frac{\partial \delta \phi_y}{\partial y} \overline{M}_{yy} + \delta \phi_y \overline{Q}_y + \delta \phi_y \left[\frac{\partial^2}{\partial t^2} \left(J_1 u_0 + K_2 \phi_x - c_1 J_4 \frac{\partial w_0}{\partial x} \right) \right] \right\} dxdy$$
$$- \oint_\Gamma \{\delta \phi_y (\overline{M}_{xy} n_x + \overline{M}_{yy} n_y)\} ds \tag{20}$$

where $\overline{V}_n$ is defined as

$$\overline{V}_n = c_1\left[\left(\frac{\partial P_{xx}}{\partial x}+\frac{\partial P_{yy}}{\partial y}\right)n_x+\left(\frac{\partial P_{xx}}{\partial x}+\frac{\partial P_{yy}}{\partial y}\right)n_y\right]-$$
$$c_1\left[\left(I_3\ddot{u}_0+J_4\ddot{\varphi}_x-c_1 I_6\frac{\partial \ddot{w}_0}{\partial x}\right)n_x+\left(I_3\ddot{v}_0+J_4\ddot{\varphi}_y-c_1 I_6\frac{\partial \ddot{w}_0}{\partial y}\right)n_y\right]+$$
$$\left(\overline{Q}_x n_x+\overline{Q}_y n_y\right)+c_1\frac{\partial P_{ns}}{\partial s} \tag{21}$$

The primary variables of the third-order theory are $u_n, u_s, w_0, \frac{\partial w_0}{\partial n}, \phi_n, \phi_s$, where (u_n, u_s) denotes in-plane normal and tangential displacements, and (ϕ_n, ϕ_s) are the rotations of a transverse line about the in-plane normal and tangent. Lagrange interpolation functions of $(u_n, u_s, \phi_n, \phi_s)$ and Hermite interpolation function for w_0 are used for the formulation of the displacement finite element model. A conforming element that has eight degrees of freedom $(u_0, v_0, w_0, w_{0,x}, w_{0,y}, w_{0,xy}, \phi_x, \phi_y)$ is used in this study.

4.3 Semidiscrete finite element model

The generalized displacements are approximated over an element Ω^e by the expressions

$$u_x(x,y,t)=\sum_{i=1}^{m} u_i^e(t)\ \psi_i^e(x,y) \tag{22}$$

$$v_x(x,y,t)=\sum_{i=1}^{m} v_i^e(t)\ \psi_i^e(x,y) \tag{23}$$

$$w_0(x,y,t)=\sum_{i=1}^{m} \overline{\Delta}_i^e(t)\ \varphi_i^e(x,y) \tag{24}$$

$$\phi_x(x,y,t)=\sum_{i=1}^{m} X_i^e(t)\ \psi_i^e(x,y) \tag{25}$$

$$\phi_y(x,y,t)=\sum_{i=1}^{m} Y_i^e(t)\ \psi_i^e(x,y) \tag{26}$$

where ψ_i^e denotes the Lagrange interpolation functions and φ_i^e are the Hermite interpolation functions. Here we chose the same approximation for the in-plane displacements (u_0, v_0) and rotations (ϕ_x, ϕ_y), although one could use different approximations for these two pairs. In the case of the conforming element, the four nodal values associated with w_0 are

$$\overline{\Delta}_1 = w_0,\ \overline{\Delta}_2 = \frac{\partial w_0}{\partial x},\ \overline{\Delta}_3 = \frac{\partial w_0}{\partial y},\ \overline{\Delta}_4 = \frac{\partial^2 w_0}{\partial x \partial y}$$

The finite element model is of the compact form

$$\sum_{\beta=1}^{5}\sum_{j=1}^{n_\beta}\left(K_{ij}^{\alpha\beta}\Delta_j^\beta + C_{ij}^{\alpha\beta}\dot{\Delta}_j^\beta + M_{ij}^{\alpha\beta}\ddot{\Delta}_j^\beta\right) - F_i^\alpha = 0 \ , \ i = 1, 2, \ldots, n \tag{27}$$

where $\alpha = 1, 2, 3, 4, 5$, $n_1 = n_2 = n_4 = n_5 = 4$ and $n_3 = 16$.

The nodal values Δ_j^β are $\Delta_j^1 = u_j$, $\Delta_j^2 = v_j$, $\Delta_j^3 = \overline{\Delta}_j$, $\Delta_j^4 = X_j$, $\Delta_j^5 = Y_j$, and damping coefficients are defined by

$$C_{ij}^{13} = \int_{\Omega^e}\left\{dc(t)\left[\sum_{k=1}^{N}(Z_{k+1} - Z_k)\right]\frac{\partial\psi_i}{\partial x}\varphi_j\right\}dxdy \tag{28}$$

$$C_{ij}^{23} = \int_{\Omega^e}\left\{dc(t)\left[\sum_{k=1}^{N}(Z_{k+1} - Z_k)\right]\frac{\partial\psi_i}{\partial y}\varphi_j\right\}dxdy \tag{29}$$

$$C_{ij}^{33} = \int_{\Omega^e}\left\{dc(t)\,(-c_1)\left[\sum_{k=1}^{N}\frac{1}{4}\left(Z_{k+1}^4 - Z_k^4\right)\right]\left(\frac{\partial^2\varphi_i}{\partial x^2} + \frac{\partial^2\varphi_i}{\partial y^2}\right)\varphi_j\right\}dxdy \tag{30}$$

$$C_{ij}^{43} = \int_{\Omega^e}\left\{dc(t)\left(\left[\sum_{k=1}^{N}\frac{1}{2}\left(Z_{k+1}^2 - Z_k^2\right)\right] - c_1\left[\sum_{k=1}^{N}\frac{1}{4}\left(Z_{k+1}^4 - Z_k^4\right)\right]\right)\frac{\partial\psi_i}{\partial x}\varphi_j\right\}dxdy \tag{31}$$

$$C_{ij}^{53} = \int_{\Omega^e}\left\{dc(t)\left(\left[\sum_{k=1}^{N}\frac{1}{2}\left(Z_{k+1}^2 - Z_k^2\right)\right] - c_1\left[\sum_{k=1}^{N}\frac{1}{4}\left(Z_{k+1}^4 - Z_k^4\right)\right]\right)\frac{\partial\psi_i}{\partial y}\varphi_j\right\}dxdy \tag{32}$$

where $dc(t) = e_m d_m k_c c(t)$ and e_m is modulus of the magnetostrictive layer, d_m the magneto-mechanical coupling coefficient, and k_c and $c(t)$ are defined in the previous section. For the details of the stiffness and mass coefficients, see Reddy [12].

This completes the general linear finite element model development of the third-order shear deformation plate theory.

4.4 Transient analysis

The equations of motion can be solved exactly using analytical methods, but those are algebraically complicated and require the determination of eignevalues and eigenfunctions, as in the state-space approach. Newmark method that takes advantage of the static solution form for spatial variation and uses a numerical method to solve the resulting differential equations in time was used to determine the transient response of composite laminates in this study.

The second-order differential equation (27) can be expressed in matrix form as

$$[M^e]\{\ddot{u}^e\} + [C^e]\{\dot{u}^e\} + [K^e]\{u^e\} = \{F^e\} \tag{33}$$

Using the Newmark's scheme, Eq. (33) can be reduced to the form

$$[\hat{K}]_{s+1}\{\Delta\}_{s+1} = \{\hat{F}\}_{s;s+1} \tag{34}$$

where

$$[\hat{K}]_{s+1} = [K]_{s+1} + a_3[M]_{s+1} + a_6[C]_{s+1} \tag{35}$$

$$\{\hat{F}\}_{s,s+1} = \{F\}_{s+1} + [M]_{s+1}\{A\}_s + [C]_{s+1}\{B\}_s \tag{36}$$

$$\{A\}_s = a_3\{u\}_s + a_4\{\dot{u}\}_s + a_5\{\ddot{u}\}_s\,,\ \{B\}_s = a_6\{u\}_s + a_7\{\dot{u}\}_s + a_8\{\ddot{u}\}_s \tag{37}$$

$$a_1 = (1-\alpha)\Delta t\,,\ a_2 = \alpha\Delta t\,,\ a_3 = \frac{2}{\gamma(\Delta t)^2}\,,\ a_4 = a_3\Delta t\,,\ a_5 = \frac{(1-\gamma)}{\gamma}\,,$$

$$a_6 = \frac{2\alpha}{\gamma\Delta t}\,,\ a_7 = \frac{2\alpha}{\gamma} - 1\,,\ a_8 = \Delta t\left(\frac{\alpha}{\gamma} - 1\right) \tag{38}$$

and α and γ are parameters that determine the stability and accuracy of the scheme.

In this numerical method, the time derivatives are approximated using difference approximations, and therefore solution is obtained only for discrete times and not as a continuous function of time.

5. NUMERICAL RESULTS AND DISCUSSION

5.1 Preliminary comments

Numerical studies are carried out to analyze the deflection suppression characteristics for different lamination schemes and boundary conditions using the developed TSDT finite element models. The baseline of the simulations is the simply supported square laminate ($a/b = 1$, $a/h = 10$; see the Figure 1) under sinusoidal distributions of the initial velocity field

$$\frac{\partial w}{\partial t}(x, y, t = 0) = \sin\frac{\pi x}{a}\sin\frac{\pi y}{b} \tag{39}$$

The time step selected in the present study is Δt = 0.0005sec. The notation for lamination scheme $(\theta_1, \theta_2, \theta_3, \theta_4, m)_s$ means that there are 10 layers symmetrically placed about the midplane with the fiber orientations being $(\theta_1, \theta_2, \theta_3, \theta_4, m, m, \theta_4, \theta_3, \theta_2, \theta_1)$, where m stands for the magnetostrictive layer and subscript s stands for symmetric. The material properties of Terfenol-D and the elastic composite materials are listed in Table 1.

In finite element analysis, solution symmetries should be taken advantage of by identifying the computational domain to reduce computational effort. For a laminated composite plate with all edges simply-supported or clamped, a quadrant of the plate may be used as the computational domain. Figure 2 shows two types of simply supported boundary conditions for the third-order shear deformation theory. Figure 3 shows the effects of the finite element results for the different laminations and boundary conditions. Quarter plate models with proper boundary conditions can be used in the antisymmetric laminates with simply supported boundary condition, but not with the clamped cases. For symmetric laminates, the simply supported cross-ply laminates can be modelled as a quarter plate. The boundary conditions along a line of symmetry should be correctly identified and imposed in the finite element model. When one is not sure of the solution symmetry, it is advised that the whole plate be modelled.

Table 1 Material properties of magnetostrictive and elastic composite materials.

Material	E_1 $[GPa]$	E_2 $[GPa]$	G_{13} $[GPa]$	G_{23} $[GPa]$	G_{12} $[GPa]$	ν_{12}	ρ $[Kg\ m^{-3}]$	D_k $[10^{-8} mA^{-1}]$
Terfenol-D	26.5	26.5	13.25	13.25	13.25	0	9250	1.67
CFRP	138.6	8.27	4.96	4.12	4.96	0.26	1824	-
Gr-Ep(AS)	137.9	8.96	7.20	6.21	7.20	0.30	1450	-
Gl-Ep	53.78	17.93	8.96	3.45	8.96	0.25	1900	-
Br-Ep	206.9	20.69	6.9	4.14	6.9	0.30	1950	-

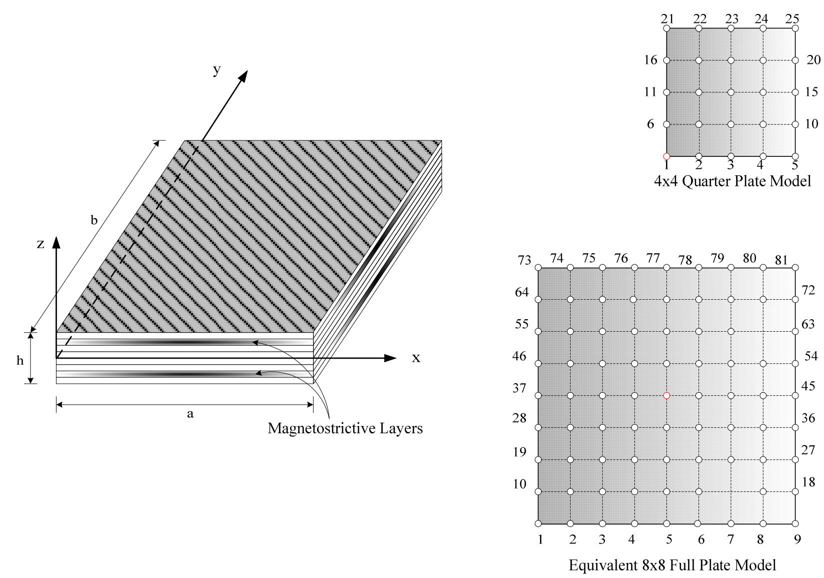


Fig. 1. Laminated composite plate configurations and finite element meshes.

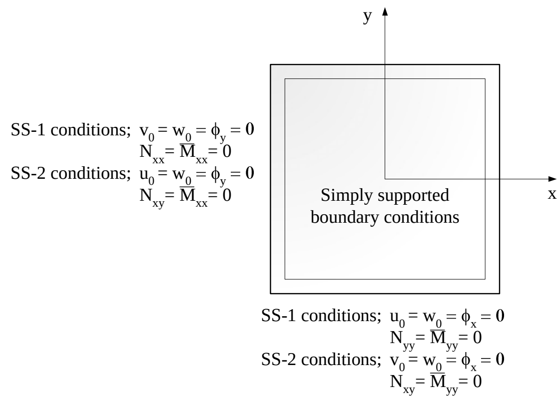
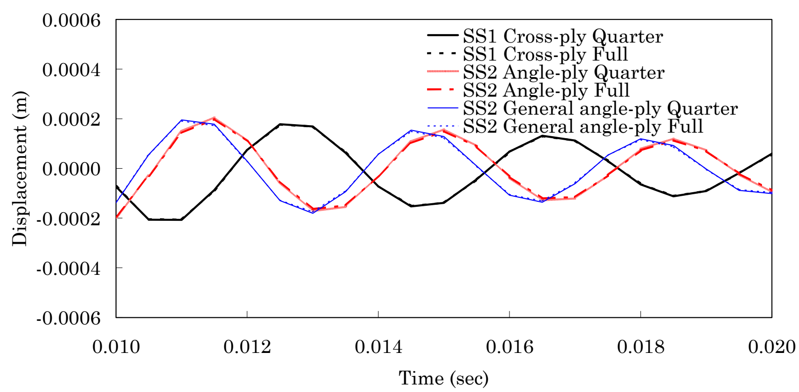
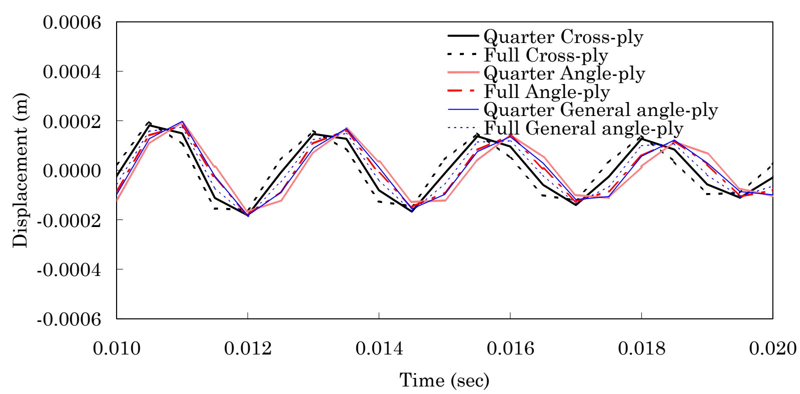


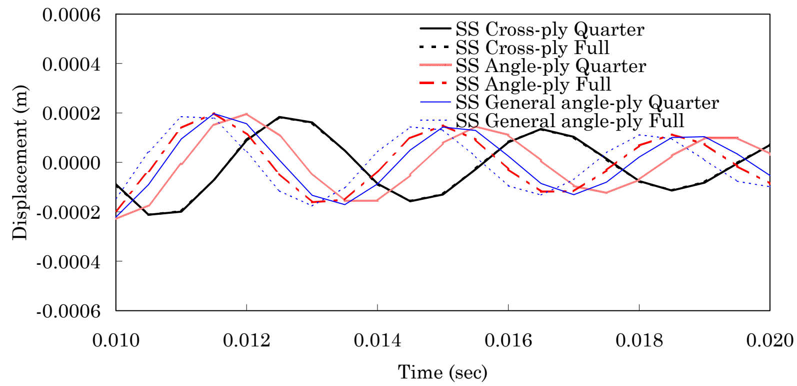
Fig. 2. Two types of simply supported boundary conditions, SS-1and SS-2, for laminated plates.



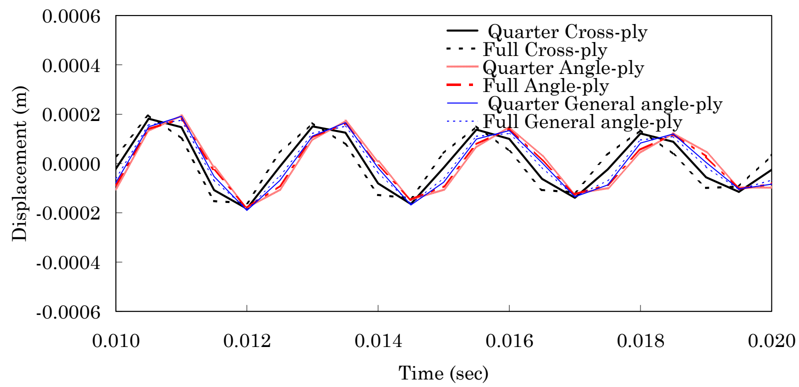
(a)



(b)



(c)



(d)

Fig. 3. Effects of using full and quarter plate models in finite element modelling of CFRP composite plates; cross-ply $(m,90,0,90,0)_{s \text{ or anti-s}}$, angle-ply $(m,30,-30,30-30)_{s \text{ or anti-s}}$, and general angle-ply $(m,45,-45,90,0)_{s \text{ or anti-s}}$;(a) Simply supported antisymmetric laminates, (b) Clamped antisymmetric laminates, (c) Simply supported symmetric laminates, (d) Clamped symmetric laminates.

5.2 Simply supported laminates

To compare with the analytical results, the SS-1 boundary conditions and quarter plate model have been used for symmetric cross-ply laminates. The deflections predicted from the analytical (eigenvalue analysis) and transient finite element analysis are within the reasonable agreement, as shown in Figure 4. Figure 5 shows the central displacements using the different plate theories (CLPT, FSDT, and TSDT) for two different lamination schemes. It is observed that CLPT shows higher deflection suppression capacity in both cases. This is expected because the CLPT renders plate stiffer compared to the other theories (also see Table 2).

After studying the influence of lamina material properties on the amplitude of vibration and vibration suppression times, it is observed that materials having the almost same E_1/E_2 ratio have similar vibration suppression characteristics. Figure 6 shows the deflection damping characteristics for the different laminate materials.

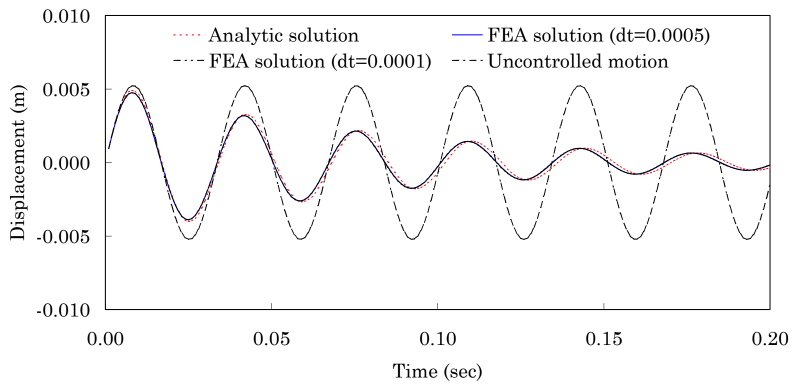


Fig. 4. Comparison of the center deflection predicted by the analytical and finite element methods for the case of symmetric cross-ply CFRP laminates $(m,90,0,90,0)_s$.

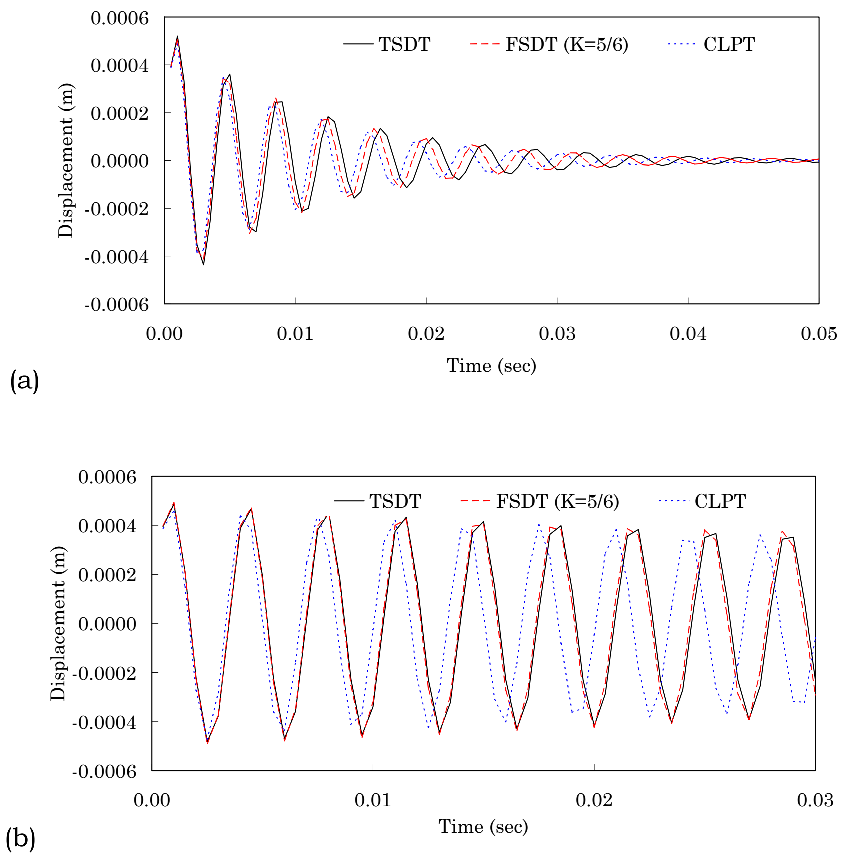


Fig. 5. Comparison of the center displacements by the different plate theories for simply supported cross-ply CFRP laminates; (a) laminate $(m,90,0,90,0)_s$, (b) laminate $(0,90,0,90,m)_s$.

Table 2 Selected center displacement values on the symmetric cross-ply CFRP laminates for the different laminate theories.

t	Center Displacement (m)					
	$(m,90,0,90,0)_s$ laminate			$(90,0,90,0,m)_s$ laminate		
	CLPT	FSDT	TSDT	CLPT	FSDT	TSDT
0.0005	3.93E-04	3.92E-04	3.89E-04	3.94E-04	4.00E-04	3.89E-04
0.0010	5.21E-04	5.06E-04	4.87E-04	4.87E-04	4.92E-04	4.57E-04
0.0015	3.32E-04	2.96E-04	2.57E-04	2.11E-04	2.09E-04	1.51E-04
0.0020	-3.72E-05	-8.25E-05	-1.20E-04	-2.21E-04	-2.30E-04	-2.74E-04
0.0025	-3.52E-04	-3.75E-04	-3.85E-04	-4.82E-04	-4.90E-04	-4.72E-04
0.0030	-4.36E-04	-4.11E-04	-3.73E-04	-3.78E-04	-3.75E-04	-2.83E-04
0.0035	-2.54E-04	-1.86E-04	-1.18E-04	1.18E-05	2.37E-05	1.35E-04
0.0040	6.08E-05	1.34E-04	1.91E-04	3.88E-04	4.01E-04	4.38E-04
0.0045	3.15E-04	3.46E-04	3.47E-04	4.68E-04	4.70E-04	3.81E-04
0.0050	3.61E-04	3.22E-04	2.61E-04	1.94E-04	1.81E-04	1.35E-05

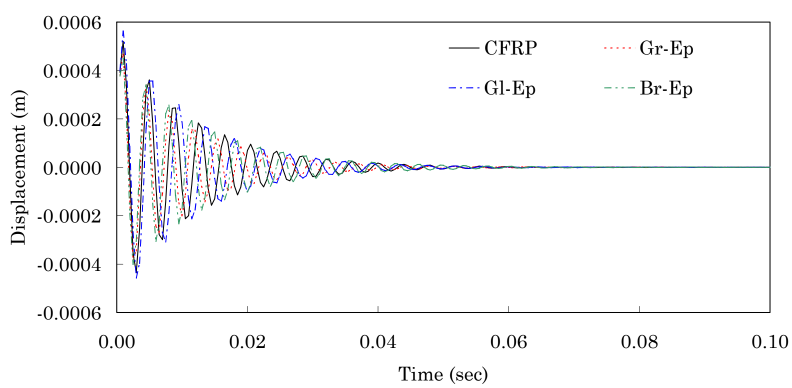


Fig. 6. The effect of the lamina material properties on the damping of deflection in symmetric cross-ply laminates $(m,90,0,90,0)_s$.

The deflection suppression time is defined as the time required to reduce the uncontrolled center deflection to one-tenth of its magnitude. The deflection suppression time ratio (suppression time divided by the maximum suppression time) can be shown to be $T_s = h_m/2z_m$, where h_m is the thickness of the magnetostrictive layer and z_m is the positive distance between the mid plane of the magnetostrictive layer and the mid plane of the plate. The maximum deflections ($W_{\max}$) of the composite plate and the suppression times for the different position of smart layers are presented in Table 3. The effect of the smart layer positions on the vibration suppression can be shown in the Figure 7. It is observed that as the smart material layer is moved farther from the mid-plane the suppression time decreases, as may be expected because of the moment effect by smart layer actuations.

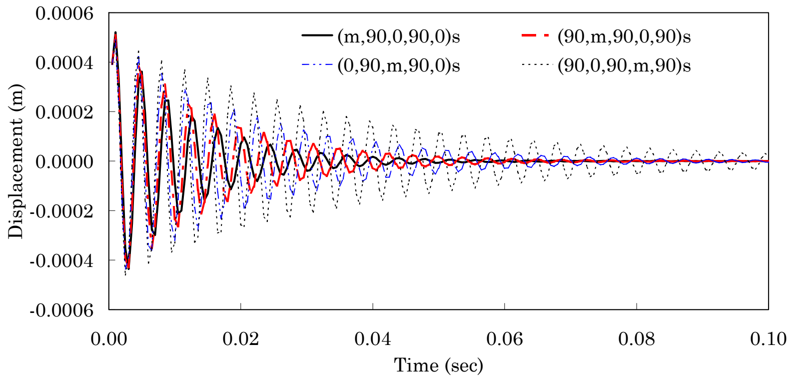


Fig. 7. The effect of the smart material layer position on the deflection for the symmetric cross-ply CFRP laminates.

Table 3 Vibration suppression time for the different smart layer positions on the symmetric cross-ply CFRP laminate $(m,90,0,90,0)_s$

Lamination Scheme	z_m (m)	T_s	W_{max} $(10^{-4} m)$	t at $W_{max}/10$
(m,90,0,90,0)s	0.045	0.111	5.21	0.0285
(90,m,90,0,90)s	0.035	0.143	5.09	0.0350
(0,90,m,90,0)s	0.025	0.200	4.90	0.0480
(90,0,90,m,90)s	0.015	0.333	4.85	0.0850
(0,90,0,90,m)s	0.005	1.000	4.87	0.2560

The effect of the thickness of smart-material layer on deflection damping characteristics is studied next. It is observed that thicker smart material layers result in better attenuation of the deflection. This is due to a larger mass inertia that is caused by the large increase in the moment of inertia of the system when thickness of the smart material layer is increased. We note that the smart material layer has a density of five times that of the composite material. The suppression times and characteristics for different smart layer thicknesses are shown in Table 4 and Figure 8.

Figure 9 shows the effect of the feedback coefficient $c(t)k_c$ on the vibration suppression characteristics. Two different values of the feedback coefficient are used; 10^4 and 10^3. It can be seen that the suppression time increases when the value of the feedback coefficient decrease. This is expected because the coefficients of the damping matrix decrease, thereby resulting in less damping.

Table 4 Suppression times for the different smart layer thicknesses in symmetric cross-ply laminates (m,90,0,90,0)$_s$

Lamina Thickness (mm)	z_m (m)	T_s	W_{max} $(10^{-4}m)$	t at $W_{max}/10$
$h_e=10,\ h_m=2$	0.0410	0.0244	4.77	0.0780
$h_e=10,\ h_m=4$	0.0420	0.0476	4.98	0.0485
$h_e=10,\ h_m=5$	0.0425	0.0588	5.05	0.0420
$h_e=10,\ h_m=6$	0.0430	0.0698	5.10	0.0350
$h_e=10,\ h_m=8$	0.0440	0.0909	5.17	0.0320
$h_e=10,\ h_m=10$	0.0450	0.1111	5.21	0.0285
$h_e=5,\ h_m=5$	0.0225	0.1111	9.12	0.0310
$h_e=5,\ h_m=10$	0.0250	0.2	9.45	0.0400

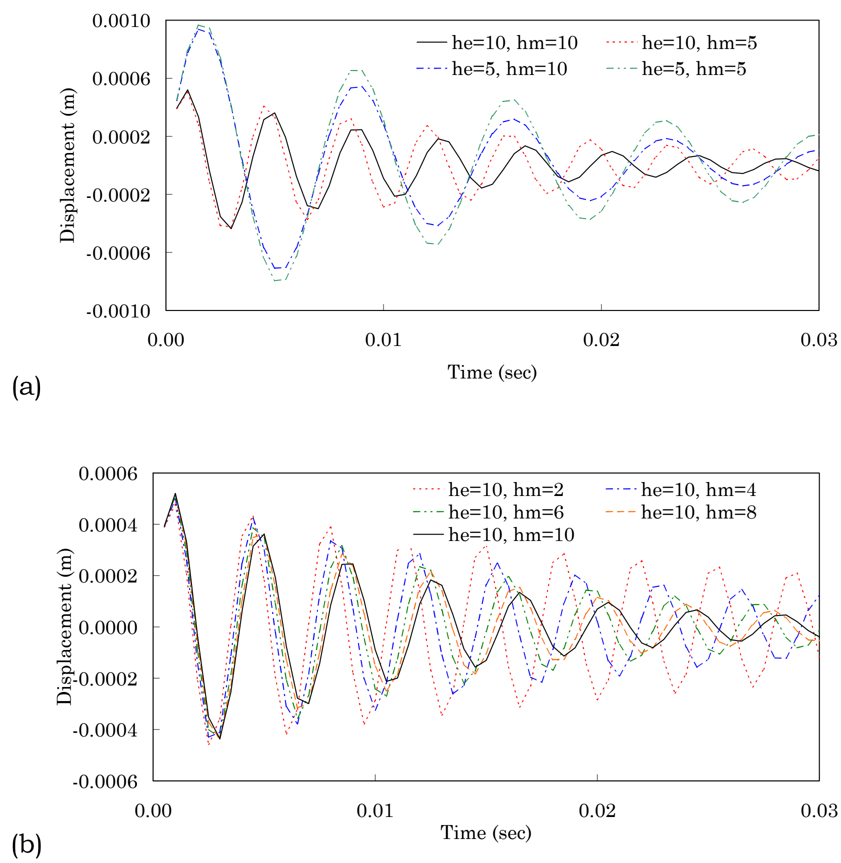


Fig. 8. The effect of the thickness of smart material layers on the deflection damping characteristics of symmetric cross-ply laminates (m,90,0,90,0)$_s$.

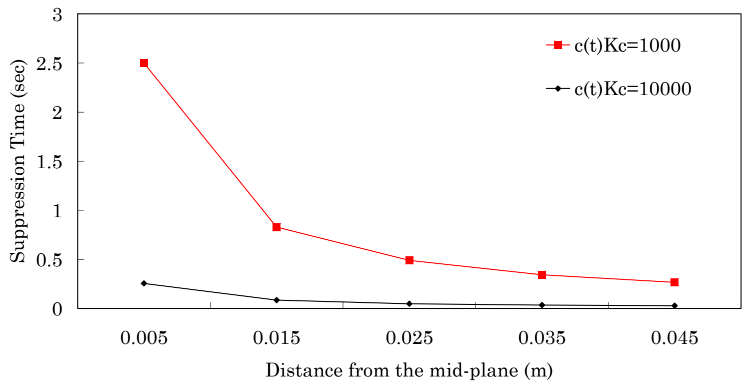
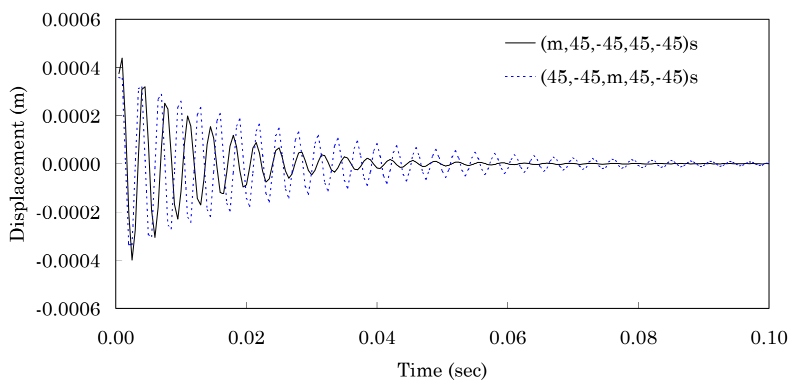
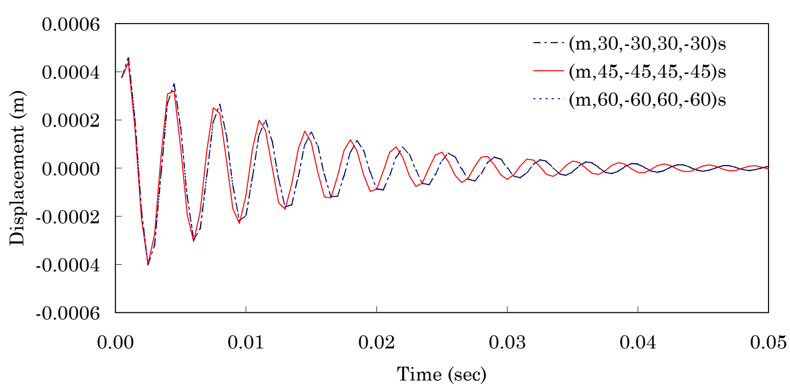


Fig. 9. Effects of the magnitude of the feedback coefficients on the suppression time for symmetric cross-ply CFRP laminates $(m,90,0,90,0)_s$.

The deflection damping characteristics of symmetric angle-ply and general angel-ply composite laminates are studied using full plate F.E. models. Observations made earlier on various characteristics such as the effects of smart layer position, its thickness, and magnitude of the feedback coefficient. are also valid for these laminates, as shown in Figures 10 and 11.



(a)



(b)

Fig. 10. Center displacement versus time for symmetric angle-ply CFRP laminates.

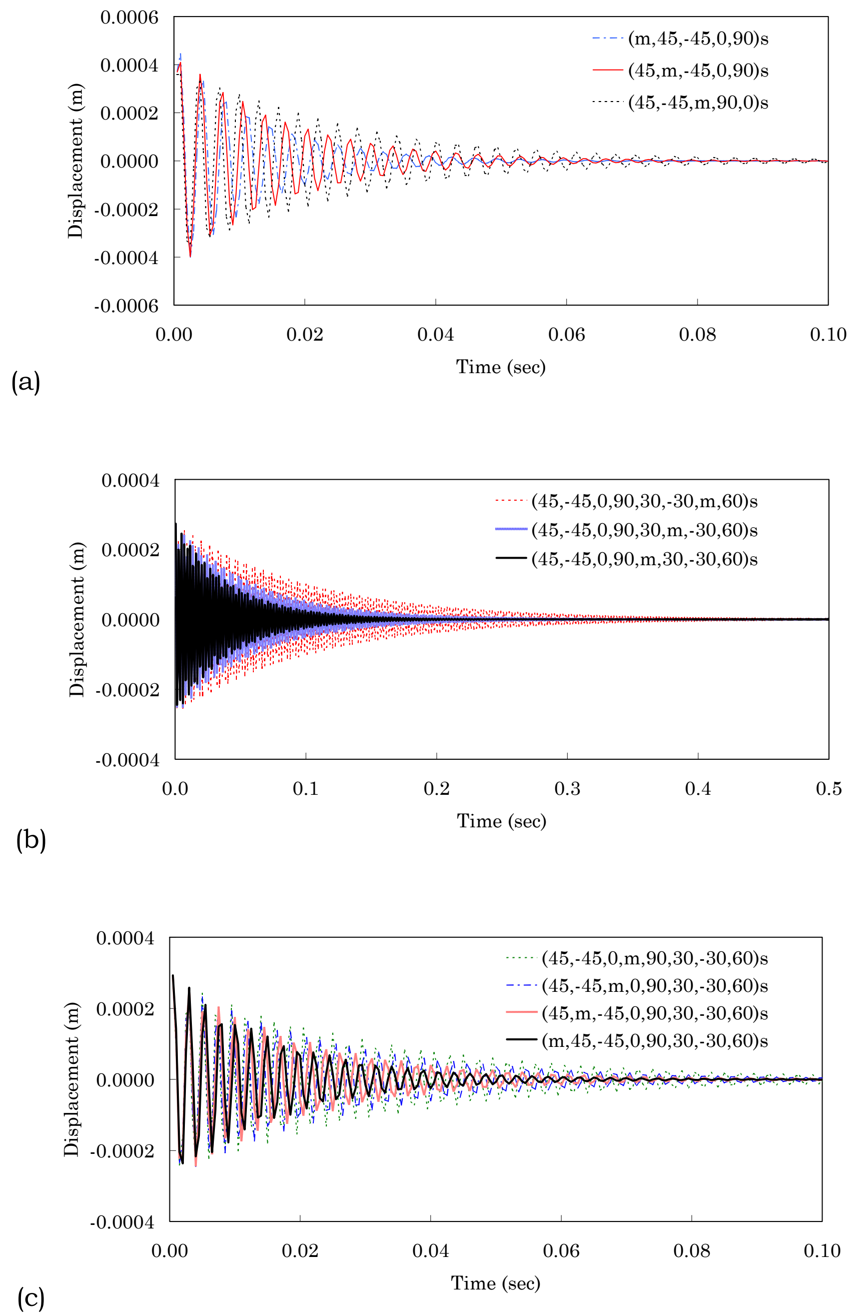
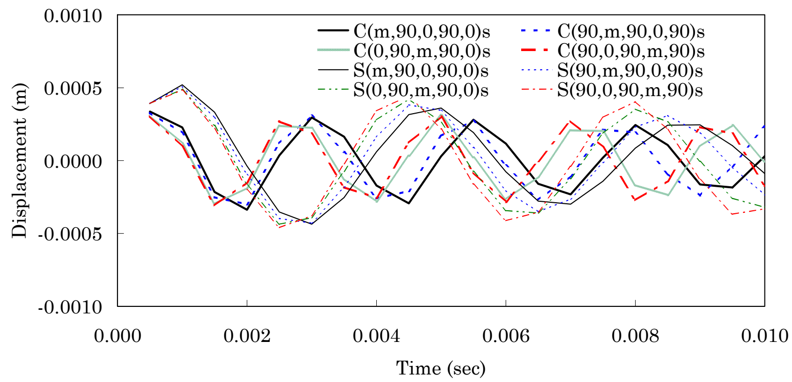


Fig. 11. The effect of the position of the smart layer on the deflection of symmetric general angle-ply CFRP laminates.

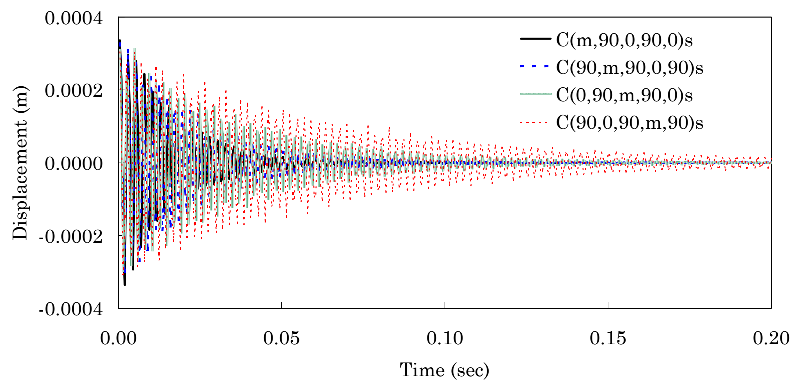
5.3 Clamped composite laminates

Next, fully clamped laminated plates are analyzed using 8×8 mesh in a full plate. The effect of the boundary conditions on the deflection is shown in the Figure 12. The maximum displacements of the simply supported plate are greater than those of the clamped case, which

is expected. Simply supported laminates, which have larger displacements, take less suppression time compared to the clamped laminates.



(a)



(b)

Fig. 12. The effect of boundary conditions on the center displacements for the different laminates; (a) Comparison of simply supported and clamped laminates, (b) results for different smart layer position.

5.4 Effects of the mechanical loading

Numerical studies are also carried out to analyze smart composite laminates under uniformly distributed load q_0 instead of specified initial velocity field. Figure 13 shows the center deflection for selected simply supported and clamped laminates under continuously applied uniformly distributed loading, while Figure 14 shows the case under suddenly applied step loading. The effect of sinusoidal loading on the central displacement has been studied. The results of symmetric cross-ply laminates with simply supported boundary conditions and subjected to sinusoidal and uniformly distributed loads are shown in Figure 15. The transverse displacements in Figures 13-15 are plotted in the nondimensionalized forms as $w_0 (100) \dfrac{E_2 h^3}{b^4 q_0}$.

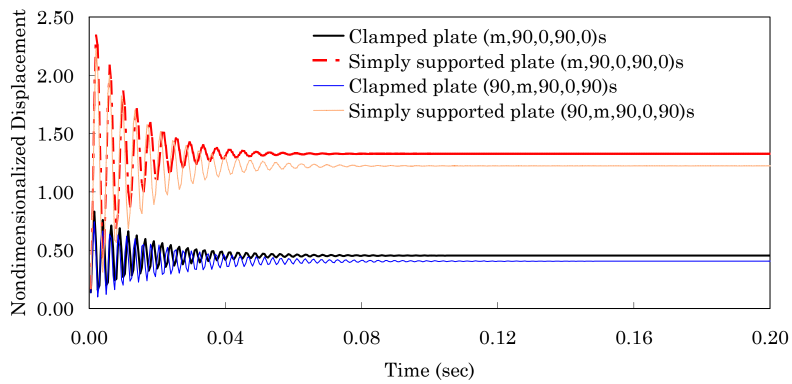


Fig. 13. Nondimensionalized center deflection versus time for simply supported and clamped laminated plates under uniform load.

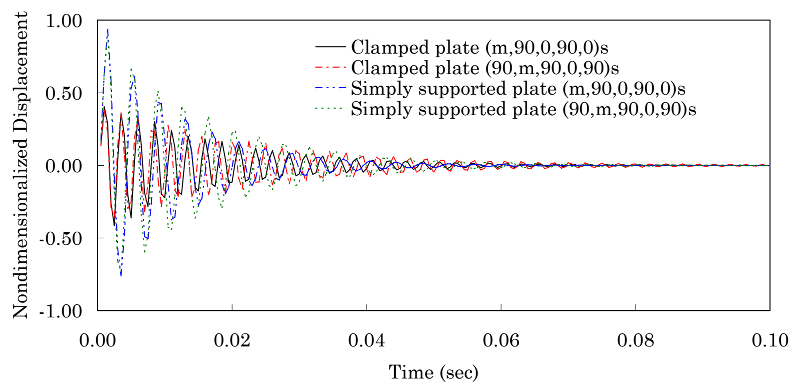


Fig. 14. Nondimensionalized center deflection versus time for simply supported and clamped laminated plates under suddenly applied uniform load.

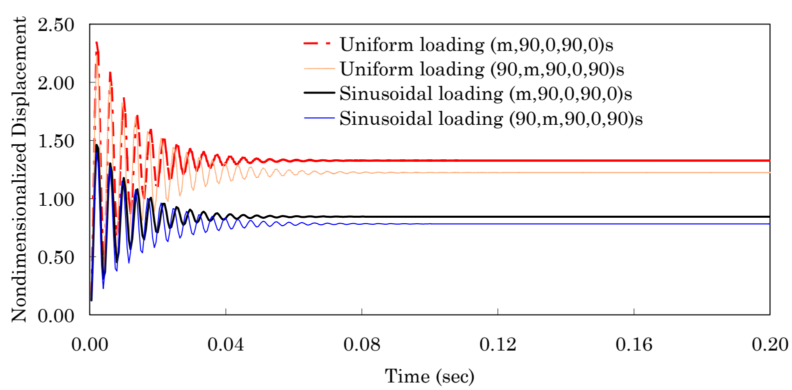


Fig. 15. Comparison of center displacement versus time for simply supported laminated plates under sinusoidal and uniformly distributed loads.

6. NONLINEAR FINITE ELEMENT MODEL

6.1 Virtual Work Statements

The weak forms of the nonlinear equations of motion are

$$0=\int_{\Omega^e}\left\{\frac{\partial\delta u_0}{\partial x}N_{xx}+\frac{\partial\delta u_0}{\partial y}N_{xy}+\delta u_0\left[I_0\frac{\partial^2 u_0}{\partial t^2}+J_1\frac{\partial^2\phi_x}{\partial t^2}-c_1I_3\frac{\partial^2}{\partial t^2}\left(\frac{\partial w_0}{\partial x}\right)\right]\right\}dxdy$$
$$-\oint_\Gamma\{\delta u_0(N_{xx}\eta_x+N_{xy}\eta_y)\}ds$$
$$0=\int_{\Omega^e}\left\{\frac{\partial\delta v_0}{\partial x}N_{xy}+\frac{\partial\delta v_0}{\partial y}N_{yy}+\delta v_0\left[I_0\frac{\partial^2 v_0}{\partial t^2}+J_1\frac{\partial^2\phi_y}{\partial t^2}-c_1I_3\frac{\partial^2}{\partial t^2}\left(\frac{\partial w_0}{\partial y}\right)\right]\right\}dxdy$$
$$-\oint_\Gamma\{\delta v_0(N_{xy}\eta_x+N_{yy}\eta_y)\}ds$$
$$0=\int_{\Omega^e}\left\{\frac{\partial\delta w_0}{\partial x}\overline{Q}_x+\frac{\partial\delta w_0}{\partial y}\overline{Q}_y-c_1\left(\frac{\partial^2\delta w_0}{\partial x^2}P_{xx}+2\frac{\partial^2\delta w_0}{\partial x\partial y}P_{xy}+\frac{\partial^2\delta w_0}{\partial y^2}p_{yy}\right)-\delta w_0 q\right.$$
$$+\frac{\partial\delta w_0}{\partial x}\left(N_{xx}\frac{\partial w_0}{\partial x}+N_{xy}\frac{\partial w_0}{\partial y}\right)+\frac{\partial\delta w_0}{\partial y}\left(N_{xy}\frac{\partial w_0}{\partial x}+N_{yy}\frac{\partial w_0}{\partial y}\right)$$
$$+\left(\delta w_0 I_0\frac{\partial^2 w_0}{\partial t^2}+c_1^2I_6\left(\frac{\partial\delta w_0}{\partial x}\frac{\partial^3 w_0}{\partial x\partial t^2}+\frac{\partial\delta w_0}{\partial y}\frac{\partial^3 w_0}{\partial y\partial t^2}\right)\right.$$
$$\left.\left.-c_1\left[I_3\left(\frac{\partial\delta w_0}{\partial x}\frac{\partial^2 u_0}{\partial t^2}+\frac{\partial\delta w_0}{\partial y}\frac{\partial^2 v_0}{\partial t^2}\right)+J_4\left(\frac{\partial\delta w_0}{\partial x}\frac{\partial^2\phi_x}{\partial t^2}+\frac{\partial\delta w_0}{\partial y}\frac{\partial^2\phi_y}{\partial t^2}\right)\right]\right)\right\}dxdy$$
$$-\oint_\Gamma\delta w_o\overline{V}_n ds-\oint_\Gamma\frac{\partial\delta w_o}{\partial n}p_{nn}ds$$
$$0=\int_{\Omega^e}\left\{\frac{\partial\delta\phi_x}{\partial x}\overline{M}_{xx}+\frac{\partial\delta\phi_x}{\partial y}\overline{M}_{xy}+\delta\phi_x\overline{Q}_x+\delta\phi_x\left[\frac{\partial^2}{\partial t^2}\left(J_1u_0+K_2\phi_x-c_1J_4\frac{\partial w_0}{\partial x}\right)\right]\right\}dxdy$$
$$-\oint_\Gamma\{\delta\phi_x(\overline{M}_{xx}\eta_x+\overline{M}_{xy}\eta_y)\}ds$$
$$0=\int_{\Omega^e}\left\{\frac{\partial\delta\phi_y}{\partial x}\overline{M}_{xy}+\frac{\partial\delta\phi_y}{\partial y}\overline{M}_{yy}+\delta\phi_y\overline{Q}_y+\delta\phi_y\left[\frac{\partial^2}{\partial t^2}\left(J_1u_0+K_2\phi_x-c_1J_4\frac{\partial w_0}{\partial x}\right)\right]\right\}dxdy$$
$$-\oint_\Gamma\{\delta\phi_y(\overline{M}_{xy}\eta_x+\overline{M}_{yy}\eta_y)\}ds \tag{40}$$

where $\overline{V}_n$ and p are defined as

$$\overline{V}_n=c_1\left[\left(\frac{\partial P_{xx}}{\partial x}+\frac{\partial P_{yy}}{\partial y}\right)n_x+\left(\frac{\partial P_{xx}}{\partial x}+\frac{\partial P_{yy}}{\partial y}\right)n_y\right]-$$
$$c_1\left[\left(I_3\ddot{u}_0+J_4\ddot{\varphi}_x-c_1I_6\frac{\partial\ddot{w}_0}{\partial x}\right)n_x+\left(I_3\ddot{v}_0+J_4\ddot{\varphi}_y-c_1I_6\frac{\partial\ddot{w}_0}{\partial y}\right)n_y\right]+$$
$$\left(\overline{Q}_xn_x+\overline{Q}_yn_y\right)+p(w_0,N_{xx},N_{yy},N_{xy})+c_1\frac{\partial P_{ns}}{\partial s}n_n$$
$$p=\left(N_{xx}\frac{\partial w_0}{\partial x}+N_{xy}\frac{\partial w_0}{\partial y}\right)n_x+\left(N_{xy}\frac{\partial w_0}{\partial x}+N_{yy}\frac{\partial w_0}{\partial y}\right)n_y \tag{41}$$

The primary variables of the third-order theory are $u_n, u_s, w_0, \frac{\partial w_0}{\partial n}, \phi_n, \phi_s$, where (u_n, u_s) denotes in-plane normal and tangential displacements, and (ϕ_n, ϕ_s) are the rotations of a transverse line about the in-plane normal and tangent. Lagrange interpolation functions of $(u_n, u_s, \phi_n, \phi_s)$ and Hermite interpolation function for w_0 are used for the formulation of the displacement finite element model. A conforming element that has eight degrees of freedom $(u_0, v_0, w_0, w_{0,x}, w_{0,y}, w_{0,xy}, \phi_x, \phi_y)$ is used.

6.2 Semidiscrete Finite Element Model

The generalized displacements are approximated over an element Ω^e by the expressions

$$
\begin{aligned}
u_x(x,y,t) &= \sum_{i=1}^{m} u_i^e(t)\ \psi_i^e(x,y) \\
v_x(x,y,t) &= \sum_{i=1}^{m} v_i^e(t)\ \psi_i^e(x,y) \\
w_0(x,y,t) &= \sum_{i=1}^{m} \overline{\Delta}_i^e(t)\ \varphi_i^e(x,y) \\
\phi_x(x,y,t) &= \sum_{i=1}^{m} X_i^e(t)\ \psi_i^e(x,y) \\
\phi_y(x,y,t) &= \sum_{i=1}^{m} Y_i^e(t)\ \psi_i^e(x,y)
\end{aligned}
\tag{42}
$$

where ψ_i^e denotes the Lagrange interpolation functions and φ_i^e are the Hermite interpolation functions. Here we chose the same approximation for the in-plane displacements (u_0, v_0) and rotations (ϕ_x, ϕ_y), although one could use different approximations for these two pairs. In the case of the conforming element, the four nodal values associated with w_0 are $\overline{\Delta}_1 = w_0,\ \overline{\Delta}_2 = \frac{\partial w_0}{\partial x},\ \overline{\Delta}_3 = \frac{\partial w_0}{\partial y},\ \overline{\Delta}_4 = \frac{\partial^2 w_0}{\partial x \partial y}$

The finite element model is of the compact form

$$
\sum_{\beta=1}^{5} \sum_{j=1}^{n_\beta} \left(K_{ij}^{\alpha\beta} \Delta_j^\beta + C_{ij}^{\alpha\beta} \dot{\Delta}_j^\beta + M_{ij}^{\alpha\beta} \ddot{\Delta}_j^\beta \right) - F_i^\alpha = 0 \ , \ i = 1, 2, \ldots, n \tag{43}
$$

where $\alpha = 1, 2, 3, 4, 5$; $n_1 = n_2 = n_4 = n_5 = 4$ and $n_3 = 16$. The nodal values Δ_j^β are $\Delta_j^1 = u_j,\ \Delta_j^2 = v_j,\ \Delta_j^3 = \overline{\Delta}_j,\ \Delta_j^4 = X_j,\ \Delta_j^5 = Y_j$, and the stiffness, mass, and damping coefficients are defined by

$$
K_{ij}^{11} = \int_{\Omega^e} \left[A_{11} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + A_{66} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + A_{16} \left(\frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) \right] dxdy
$$

$$
K_{ij}^{12} = \int_{\Omega^e} \left(A_{12} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + A_{16} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + A_{26} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + A_{66} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) dxdy
$$

$$K_{ij}^{13} = \int_{\Omega^e} \left\{ (-c_1) \left[\left(E_{11} \frac{\partial \psi_i}{\partial x} \frac{\partial^2 \varphi_j}{\partial x^2} + E_{12} \frac{\partial \psi_i}{\partial x} \frac{\partial^2 \varphi_j}{\partial y^2} + 2E_{16} \frac{\partial \psi_i}{\partial x} \frac{\partial^2 \varphi_j}{\partial x \partial y} \right) + \left(E_{16} \frac{\partial \psi_i}{\partial y} \frac{\partial^2 \varphi_j}{\partial x^2} + E_{26} \frac{\partial \psi_i}{\partial y} \frac{\partial^2 \varphi_j}{\partial y^2} + 2E_{66} \frac{\partial \psi_i}{\partial y} \frac{\partial^2 \varphi_j}{\partial x \partial y} \right) \right] \right\} dxdy$$

$$(K_{NL}^{13})_{ij} = \int_{\Omega^e} \left\{ \frac{1}{2} \left[\left(A_{11} \frac{\partial w_0}{\partial x} \frac{\partial \psi_i}{\partial x} \frac{\partial \varphi_j}{\partial x} + A_{12} \frac{\partial w_0}{\partial y} \frac{\partial \psi_i}{\partial x} \frac{\partial \varphi_j}{\partial y} + A_{16} \left(\frac{\partial w_0}{\partial x} \frac{\partial \psi_i}{\partial x} \frac{\partial \varphi_j}{\partial y} + \frac{\partial w_0}{\partial y} \frac{\partial \psi_i}{\partial x} \frac{\partial \varphi_j}{\partial x} \right) \right) + \left(A_{16} \frac{\partial w_0}{\partial x} \frac{\partial \psi_i}{\partial y} \frac{\partial \varphi_j}{\partial x} + A_{26} \frac{\partial w_0}{\partial y} \frac{\partial \psi_i}{\partial y} \frac{\partial \varphi_j}{\partial y} + A_{66} \left(\frac{\partial w_0}{\partial x} \frac{\partial \psi_i}{\partial y} \frac{\partial \varphi_j}{\partial y} + \frac{\partial w_0}{\partial y} \frac{\partial \psi_i}{\partial y} \frac{\partial \varphi_j}{\partial x} \right) \right) \right] \right\} dxdy$$

$$K_{ij}^{14} = \int_{\Omega^e} \left\{ \left[B_{11} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + B_{66} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + B_{16} \left(\frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) \right] + (-c_1) \left[E_{11} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + E_{66} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + E_{16} \left(\frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) \right] \right\} dxdy$$

$$K_{ij}^{15} = \int_{\Omega^e} \left\{ \left(B_{12} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + B_{16} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + B_{26} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + B_{66} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) + (-c_1) \left(E_{12} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + E_{16} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + E_{26} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + E_{66} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) \right\} dxdy$$

$$K_{ij}^{21} = \int_{\Omega^e} \left(A_{16} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + A_{66} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + A_{12} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} + A_{26} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) dxdy$$

$$K_{ij}^{22} = \int_{\Omega^e} \left[A_{22} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + A_{66} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + A_{26} \left(\frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) \right] dxdy$$

$$K_{ij}^{23} = \int_{\Omega^e} \left\{ (-c_1) \left[\left(E_{12} \frac{\partial \psi_i}{\partial y} \frac{\partial^2 \varphi_j}{\partial x^2} + E_{22} \frac{\partial \psi_i}{\partial y} \frac{\partial^2 \varphi_j}{\partial y^2} + 2E_{26} \frac{\partial \psi_i}{\partial y} \frac{\partial^2 \varphi_j}{\partial x \partial y} \right) + \left(E_{16} \frac{\partial \psi_i}{\partial x} \frac{\partial^2 \varphi_j}{\partial x^2} + E_{26} \frac{\partial \psi_i}{\partial x} \frac{\partial^2 \varphi_j}{\partial y^2} + 2E_{66} \frac{\partial \psi_i}{\partial x} \frac{\partial^2 \varphi_j}{\partial x \partial y} \right) \right] \right\} dxdy$$

$$(K_{NL}^{23})_{ij} = \int_{\Omega^e} \left\{ \frac{1}{2} \left[\left(A_{12} \frac{\partial w_0}{\partial x} \frac{\partial \psi_i}{\partial y} \frac{\partial \varphi_j}{\partial x} + A_{22} \frac{\partial w_0}{\partial y} \frac{\partial \psi_i}{\partial y} \frac{\partial \varphi_j}{\partial y} + A_{26} \left(\frac{\partial w_0}{\partial x} \frac{\partial \psi_i}{\partial y} \frac{\partial \varphi_j}{\partial y} + \frac{\partial w_0}{\partial y} \frac{\partial \psi_i}{\partial y} \frac{\partial \varphi_j}{\partial x} \right) \right) + \left(A_{16} \frac{\partial w_0}{\partial x} \frac{\partial \psi_i}{\partial x} \frac{\partial \varphi_j}{\partial x} + A_{26} \frac{\partial w_0}{\partial y} \frac{\partial \psi_i}{\partial x} \frac{\partial \varphi_j}{\partial y} + A_{66} \left(\frac{\partial w_0}{\partial x} \frac{\partial \psi_i}{\partial x} \frac{\partial \varphi_j}{\partial y} + \frac{\partial w_0}{\partial y} \frac{\partial \psi_i}{\partial x} \frac{\partial \varphi_j}{\partial x} \right) \right) \right] \right\} dxdy$$

$$K_{ij}^{24} = \int_{\Omega^e} \left\{ \left(B_{16} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + B_{66} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + B_{12} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} + B_{26} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) \right.$$

$$\left. + (-c_1) \left(E_{16} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + E_{66} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + E_{12} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} + E_{26} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) \right\} dxdy$$

$$K_{ij}^{25} = \int_{\Omega^e} \left\{ \left[B_{22} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + B_{66} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + B_{26} \left(\frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) \right] \right.$$

$$\left. + (-c_1) \left[E_{22} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + E_{66} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + E_{26} \left(\frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) \right] \right\} dxdy$$

$$K_{ij}^{31} = \int_{\Omega^e} -c_1 \left[\frac{\partial^2 \varphi_i}{\partial x^2} \left(E_{11} \frac{\partial \psi_j}{\partial x} + E_{16} \frac{\partial \psi_j}{\partial y} \right) + 2 \frac{\partial^2 \varphi_i}{\partial x \partial y} \left(E_{16} \frac{\partial \psi_j}{\partial x} + E_{66} \frac{\partial \psi_j}{\partial y} \right) \right.$$

$$\left. + \frac{\partial^2 \varphi_i}{\partial y^2} \left(E_{12} \frac{\partial \psi_j}{\partial x} + E_{26} \frac{\partial \psi_j}{\partial y} \right) \right] dxdy$$

$$(K_{NL}^{31})_{ij} = \int_{\Omega^e} \left\{ \frac{\partial \psi_i}{\partial x} \left[\frac{\partial w_0}{\partial x} \left(A_{11} \frac{\partial \psi_j}{\partial x} + A_{16} \frac{\partial \psi_j}{\partial y} \right) + \frac{\partial w_0}{\partial y} \left(A_{16} \frac{\partial \psi_j}{\partial x} + A_{66} \frac{\partial \psi_j}{\partial y} \right) \right] \right.$$

$$\left. + \frac{\partial \psi_i}{\partial y} \left[\frac{\partial w_0}{\partial x} \left(A_{16} \frac{\partial \psi_j}{\partial x} + A_{66} \frac{\partial \psi_j}{\partial y} \right) + \frac{\partial w_0}{\partial y} \left(A_{12} \frac{\partial \psi_j}{\partial x} + A_{26} \frac{\partial \psi_j}{\partial y} \right) \right] \right\} dady$$

$$K_{ij}^{32} = \int_{\Omega^e} -c_1 \left[\frac{\partial^2 \varphi_i}{\partial x^2} \left(E_{12} \frac{\partial \psi_j}{\partial y} + E_{16} \frac{\partial \psi_j}{\partial x} \right) + 2 \frac{\partial^2 \varphi_i}{\partial x \partial y} \left(E_{26} \frac{\partial \psi_j}{\partial y} + E_{66} \frac{\partial \psi_j}{\partial x} \right) \right.$$

$$\left. + \frac{\partial^2 \varphi_i}{\partial y^2} \left(E_{22} \frac{\partial \psi_j}{\partial y} + E_{26} \frac{\partial \psi_j}{\partial x} \right) \right] dxdy$$

$$(K_{NL}^{32})_{ij} = \int_{\Omega^e} \left\{ \frac{\partial \psi_i}{\partial x} \left[\frac{\partial w_0}{\partial x} \left(A_{12} \frac{\partial \psi_j}{\partial y} + A_{16} \frac{\partial \psi_j}{\partial x} \right) + \frac{\partial w_0}{\partial y} \left(A_{26} \frac{\partial \psi_j}{\partial y} + A_{66} \frac{\partial \psi_j}{\partial x} \right) \right] \right.$$

$$\left. + \frac{\partial \psi_i}{\partial y} \left[\frac{\partial w_0}{\partial x} \left(A_{26} \frac{\partial \psi_j}{\partial y} + A_{66} \frac{\partial \psi_j}{\partial x} \right) + \frac{\partial w_0}{\partial y} \left(A_{22} \frac{\partial \psi_j}{\partial y} + A_{26} \frac{\partial \psi_j}{\partial x} \right) \right] \right\} dady$$

$$K_{ij}^{33} = \int_{\Omega^e} \left\{ \frac{\partial \varphi_i}{\partial x} \left[\overline{A}_{45} \frac{\partial \varphi_j}{\partial y} + \overline{A}_{55} \frac{\partial \varphi_j}{\partial x} - c_2 \left(\overline{D}_{45} \frac{\partial \varphi_j}{\partial y} + \overline{D}_{55} \frac{\partial \varphi_j}{\partial x} \right) \right] \right.$$

$$+ \frac{\partial \varphi_i}{\partial y} \left[\overline{A}_{44} \frac{\partial \varphi_j}{\partial y} + \overline{A}_{45} \frac{\partial \varphi_j}{\partial x} - c_2 \left(\overline{D}_{44} \frac{\partial \varphi_j}{\partial y} + \overline{D}_{45} \frac{\partial \varphi_j}{\partial x} \right) \right]$$

$$+ (c_1)^2 \left[\frac{\partial^2 \varphi_i}{\partial x^2} \left(H_{11} \frac{\partial^2 \varphi_j}{\partial x^2} + H_{12} \frac{\partial^2 \varphi_j}{\partial y^2} + 2H_{16} \frac{\partial^2 \varphi_j}{\partial x \partial y} \right) + 2 \frac{\partial^2 \varphi_i}{\partial x \partial y} \left(H_{16} \frac{\partial^2 \varphi_j}{\partial x^2} + H_{26} \frac{\partial^2 \varphi_j}{\partial y^2} + 2H_{66} \frac{\partial^2 \varphi_j}{\partial x \partial y} \right) \right.$$

$$\left. \left. + \frac{\partial^2 \varphi_i}{\partial^2 y} \left(H_{12} \frac{\partial^2 \varphi_j}{\partial x^2} + H_{22} \frac{\partial^2 \varphi_j}{\partial y^2} + 2H_{26} \frac{\partial^2 \varphi_j}{\partial x \partial y} \right) \right] \right\} dxdy$$

$$(K_{NL}^{33})_{ij}^{(1)} = \int_{\Omega^e} - c_1 \left(\frac{1}{2} \right) \left\{ \frac{\partial^2 \varphi_i}{\partial x^2} \left[E_{11} \left(\frac{\partial w_0}{\partial x} \right) \left(\frac{\partial \varphi_j}{\partial x} \right) + E_{12} \left(\frac{\partial w_0}{\partial y} \right) \left(\frac{\partial \varphi_j}{\partial y} \right) + E_{16} \left(\frac{\partial w_0}{\partial x} \frac{\partial \varphi_j}{\partial y} + \frac{\partial \varphi_j}{\partial x} \frac{\partial w_0}{\partial y} \right) \right] \right.$$

$$+ 2 \frac{\partial^2 \varphi_i}{\partial x \partial y} \left[E_{16} \left(\frac{\partial w_0}{\partial x} \right) \left(\frac{\partial \varphi_j}{\partial x} \right) + E_{26} \left(\frac{\partial w_0}{\partial y} \right) \left(\frac{\partial \varphi_j}{\partial y} \right) + E_{66} \left(\frac{\partial w_0}{\partial x} \frac{\partial \varphi_j}{\partial y} + \frac{\partial \varphi_j}{\partial x} \frac{\partial w_0}{\partial y} \right) \right]$$

$$\left. + \frac{\partial^2 \varphi_i}{\partial^2 y} \left[E_{12} \left(\frac{\partial w_0}{\partial x} \right) \left(\frac{\partial \varphi_j}{\partial x} \right) + E_{22} \left(\frac{\partial w_0}{\partial y} \right) \left(\frac{\partial \varphi_j}{\partial y} \right) + E_{26} \left(\frac{\partial w_0}{\partial x} \frac{\partial \varphi_j}{\partial y} + \frac{\partial \varphi_j}{\partial x} \frac{\partial w_0}{\partial y} \right) \right] \right\} dxdy$$

$$(K_{NL}^{33})_{ij}^{(2)} = \left(\frac{1}{2} \right) \int_{\Omega^e} \left\{ \frac{\partial \psi_i}{\partial x} \left[\left\{ \left[A_{11} \left(\frac{\partial w_0}{\partial x} \right)^2 \frac{\partial \varphi_j}{\partial x} + A_{12} \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \frac{\partial \varphi_j}{\partial y} + A_{16} \left(\left(\frac{\partial w_0}{\partial x} \right)^2 \frac{\partial \varphi_j}{\partial y} + \frac{\partial \varphi_j}{\partial x} \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \right) \right] \right. \right. \right.$$

$$\left. - c_1 \frac{\partial w_0}{\partial x} \left(E_{11} \frac{\partial^2 \varphi_j}{\partial x^2} + E_{12} \frac{\partial^2 \varphi_j}{\partial y^2} + 2E_{16} \frac{\partial^2 \varphi_j}{\partial x \partial y} \right) \right\} +$$

$$\left\{ \left[A_{16} \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \frac{\partial \varphi_j}{\partial x} + A_{26} \left(\frac{\partial w_0}{\partial y} \right)^2 \frac{\partial \varphi_j}{\partial y} + A_{66} \left(\frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \frac{\partial \varphi_j}{\partial y} + \frac{\partial \varphi_j}{\partial x} \left(\frac{\partial w_0}{\partial y} \right)^2 \right) \right] \right.$$

$$\left. \left. - c_1 \frac{\partial w_0}{\partial y} \left(E_{16} \frac{\partial^2 \varphi_j}{\partial x^2} + E_{26} \frac{\partial^2 \varphi_j}{\partial y^2} + 2E_{66} \frac{\partial^2 \varphi_j}{\partial x \partial y} \right) \right\} \right] +$$

$$\frac{\partial \psi_i}{\partial y} \left[\left\{ \left[A_{16} \left(\frac{\partial w_0}{\partial x} \right)^2 \frac{\partial \varphi_j}{\partial x} + A_{26} \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \frac{\partial \varphi_j}{\partial y} + A_{66} \left(\left(\frac{\partial w_0}{\partial x} \right)^2 \frac{\partial \varphi_j}{\partial y} + \frac{\partial \varphi_j}{\partial x} \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \right) \right] \right. \right.$$

$$\left. - c_1 \frac{\partial w_0}{\partial x} \left(E_{16} \frac{\partial^2 \varphi_j}{\partial x^2} + E_{26} \frac{\partial^2 \varphi_j}{\partial y^2} + 2E_{66} \frac{\partial^2 \varphi_j}{\partial x \partial y} \right) \right\} +$$

$$\left\{ \left[A_{12} \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \frac{\partial \varphi_j}{\partial x} + A_{22} \left(\frac{\partial w_0}{\partial y} \right)^2 \frac{\partial \varphi_j}{\partial y} + A_{26} \left(\frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \frac{\partial \varphi_j}{\partial y} + \frac{\partial \varphi_j}{\partial x} \left(\frac{\partial w_0}{\partial y} \right)^2 \right) \right] \right.$$

$$\left. \left. \left. - c_1 \frac{\partial w_0}{\partial y} \left(E_{12} \frac{\partial^2 \varphi_j}{\partial x^2} + E_{22} \frac{\partial^2 \varphi_j}{\partial y^2} + 2E_{26} \frac{\partial^2 \varphi_j}{\partial x \partial y} \right) \right\} \right] \right\} dxdy$$

$$K_{ij}^{34} = \int_{\Omega^e} \left\{ \frac{\partial \varphi_i}{\partial x}\left(\overline{A}_{55}\psi_j - c_2\overline{D}_{55}\psi_j\right) + \frac{\partial \varphi_i}{\partial y}\left(\overline{A}_{45}\psi_j - c_2\overline{D}_{45}\psi_j\right)\right.$$

$$+(-c_1)\left[\frac{\partial^2 \varphi_j}{\partial x^2}\left(F_{11}\frac{\partial \psi_j}{\partial x} + F_{16}\frac{\partial \psi_j}{\partial y} - c_1\left(H_{11}\frac{\partial \psi_j}{\partial x} + H_{16}\frac{\partial \psi_j}{\partial y}\right)\right)\right.$$

$$+2\frac{\partial^2 \varphi_j}{\partial x \partial y}\left(F_{16}\frac{\partial \psi_j}{\partial x} + F_{66}\frac{\partial \psi_j}{\partial y} - c_1\left(H_{16}\frac{\partial \psi_j}{\partial x} + H_{66}\frac{\partial \psi_j}{\partial y}\right)\right)$$

$$\left.\left.+\frac{\partial^2 \varphi_j}{\partial^2 y}\left(F_{12}\frac{\partial \psi_j}{\partial x} + F_{26}\frac{\partial \psi_j}{\partial y} - c_1\left(H_{12}\frac{\partial \psi_j}{\partial x} + H_{26}\frac{\partial \psi_j}{\partial y}\right)\right)\right]\right\} dxdy$$

$$(K_{NL}^{34})_{ij} = \int_{\Omega^e} \left\{ \frac{\partial \psi_i}{\partial x}\left[\frac{\partial w_0}{\partial x}\left[\left(B_{11}\frac{\partial \psi_j}{\partial x} + B_{16}\frac{\partial \psi_j}{\partial y}\right) - c_1\left(E_{11}\frac{\partial \psi_j}{\partial x} + E_{16}\frac{\partial \psi_j}{\partial y}\right)\right]\right.\right.$$

$$\left.+\frac{\partial w_0}{\partial y}\left[\left(B_{16}\frac{\partial \psi_j}{\partial x} + B_{66}\frac{\partial \psi_j}{\partial y}\right) - c_1\left(E_{16}\frac{\partial \psi_j}{\partial x} + E_{66}\frac{\partial \psi_j}{\partial y}\right)\right]\right]$$

$$+\frac{\partial \psi_i}{\partial y}\left[\frac{\partial w_0}{\partial x}\left[\left(B_{16}\frac{\partial \psi_j}{\partial x} + B_{66}\frac{\partial \psi_j}{\partial y}\right) - c_1\left(E_{16}\frac{\partial \psi_j}{\partial x} + E_{66}\frac{\partial \psi_j}{\partial y}\right)\right]\right.$$

$$\left.\left.+\frac{\partial w_0}{\partial y}\left[\left(B_{12}\frac{\partial \psi_j}{\partial x} + B_{26}\frac{\partial \psi_j}{\partial y}\right) - c_1\left(E_{12}\frac{\partial \psi_j}{\partial x} + E_{26}\frac{\partial \psi_j}{\partial y}\right)\right]\right]\right\} dxdy$$

$$K_{ij}^{35} = \int_{\Omega^e} \left\{ \frac{\partial \varphi_i}{\partial x}\left(\overline{A}_{45}\psi_j - c_2\overline{D}_{45}\psi_j\right) + \frac{\partial \varphi_i}{\partial y}\left(\overline{A}_{44}\psi_j - c_2\overline{D}_{44}\psi_j\right)\right.$$

$$+(-c_1)\left[\frac{\partial^2 \varphi_j}{\partial x^2}\left(F_{12}\frac{\partial \psi_j}{\partial y} + F_{16}\frac{\partial \psi_j}{\partial x} - c_1\left(H_{12}\frac{\partial \psi_j}{\partial y} + H_{16}\frac{\partial \psi_j}{\partial x}\right)\right)\right.$$

$$+2\frac{\partial^2 \varphi_j}{\partial x \partial y}\left(F_{26}\frac{\partial \psi_j}{\partial y} + F_{66}\frac{\partial \psi_j}{\partial x} - c_1\left(H_{26}\frac{\partial \psi_j}{\partial y} + H_{66}\frac{\partial \psi_j}{\partial x}\right)\right)$$

$$\left.\left.+\frac{\partial^2 \varphi_j}{\partial^2 y}\left(F_{22}\frac{\partial \psi_j}{\partial y} + F_{26}\frac{\partial \psi_j}{\partial x} - c_1\left(H_{22}\frac{\partial \psi_j}{\partial y} + H_{26}\frac{\partial \psi_j}{\partial x}\right)\right)\right]\right\} dxdy$$

$$(K_{NL}^{35})_{ij} = \int_{\Omega^e} \left\{ \frac{\partial \psi_i}{\partial x}\left[\frac{\partial w_0}{\partial x}\left[\left(B_{12}\frac{\partial \psi_j}{\partial y} + B_{16}\frac{\partial \psi_j}{\partial x}\right) - c_1\left(E_{12}\frac{\partial \psi_j}{\partial y} + E_{16}\frac{\partial \psi_j}{\partial x}\right)\right]\right.\right.$$

$$\left.+\frac{\partial w_0}{\partial y}\left[\left(B_{26}\frac{\partial \psi_j}{\partial y} + B_{66}\frac{\partial \psi_j}{\partial x}\right) - c_1\left(E_{26}\frac{\partial \psi_j}{\partial y} + E_{66}\frac{\partial \psi_j}{\partial x}\right)\right]\right]$$

$$+\frac{\partial \psi_i}{\partial y}\left[\frac{\partial w_0}{\partial x}\left[\left(B_{26}\frac{\partial \psi_j}{\partial y} + B_{66}\frac{\partial \psi_j}{\partial x}\right) - c_1\left(E_{26}\frac{\partial \psi_j}{\partial y} + E_{66}\frac{\partial \psi_j}{\partial x}\right)\right]\right.$$

$$\left.\left.+\frac{\partial w_0}{\partial y}\left[\left(B_{22}\frac{\partial \psi_j}{\partial y} + B_{26}\frac{\partial \psi_j}{\partial x}\right) - c_1\left(E_{22}\frac{\partial \psi_j}{\partial y} + E_{26}\frac{\partial \psi_j}{\partial x}\right)\right]\right]\right\} dxdy$$

$$K_{ij}^{41} = \int_{\Omega^e} \left[\hat{B}_{11} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \hat{B}_{66} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + \hat{B}_{16} \left(\frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) \right] dxdy$$

$$K_{ij}^{42} = \int_{\Omega^e} \left(\hat{B}_{12} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + \hat{B}_{16} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \hat{B}_{26} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + \hat{B}_{66} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) dxdy$$

$$\begin{aligned} K_{ij}^{43} = \int_{\Omega^e} \Bigg\{ (-c_1) \Bigg[& \left(\hat{F}_{11} \frac{\partial \psi_i}{\partial x} \frac{\partial^2 \varphi_j}{\partial x^2} + \hat{F}_{12} \frac{\partial \psi_i}{\partial x} \frac{\partial^2 \varphi_j}{\partial y^2} + 2\hat{F}_{16} \frac{\partial \psi_i}{\partial x} \frac{\partial^2 \varphi_j}{\partial x \partial y} \right) \\ & + \left[\overline{A}_{45} \psi_i \frac{\partial \varphi_j}{\partial y} + \overline{A}_{55} \psi_i \frac{\partial \varphi_j}{\partial x} - c_2 \left(\overline{D}_{45} \psi_i \frac{\partial \varphi_j}{\partial y} + \overline{D}_{55} \psi_i \frac{\partial \varphi_j}{\partial x} \right) \right] \\ & + \left(\hat{F}_{16} \frac{\partial \psi_i}{\partial y} \frac{\partial^2 \varphi_j}{\partial x^2} + \hat{F}_{26} \frac{\partial \psi_i}{\partial y} \frac{\partial^2 \varphi_j}{\partial y^2} + 2\hat{F}_{66} \frac{\partial \psi_i}{\partial y} \frac{\partial^2 \varphi_j}{\partial x \partial y} \right) \Bigg] \Bigg\} dxdy \end{aligned}$$

$$\begin{aligned} (K_{NL}^{43})_{ij} = \int_{\Omega^e} \Bigg\{ \frac{1}{2} \Bigg[& \left(\hat{B}_{11} \frac{\partial w_0}{\partial x} \frac{\partial \psi_i}{\partial x} \frac{\partial \varphi_j}{\partial x} + \hat{B}_{12} \frac{\partial w_0}{\partial y} \frac{\partial \psi_i}{\partial x} \frac{\partial \varphi_j}{\partial y} + \hat{B}_{16} \left(\frac{\partial w_0}{\partial x} \frac{\partial \psi_i}{\partial x} \frac{\partial \varphi_j}{\partial y} + \frac{\partial w_0}{\partial y} \frac{\partial \psi_i}{\partial x} \frac{\partial \varphi_j}{\partial x} \right) \right) \\ & + \left(\hat{B}_{16} \frac{\partial w_0}{\partial x} \frac{\partial \psi_i}{\partial y} \frac{\partial \varphi_j}{\partial x} + \hat{B}_{26} \frac{\partial w_0}{\partial y} \frac{\partial \psi_i}{\partial y} \frac{\partial \varphi_j}{\partial y} + \hat{B}_{66} \left(\frac{\partial w_0}{\partial x} \frac{\partial \psi_i}{\partial y} \frac{\partial \varphi_j}{\partial y} + \frac{\partial w_0}{\partial y} \frac{\partial \psi_i}{\partial y} \frac{\partial \varphi_j}{\partial x} \right) \right) \Bigg] \Bigg\} dxdy \end{aligned}$$

$$\begin{aligned} K_{ij}^{44} = \int_{\Omega^e} \Bigg\{ & \left[\hat{D}_{11} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \hat{D}_{66} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + \hat{D}_{16} \left(\frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) \right] \\ & + \left(\overline{A}_{55} \psi_i \psi_j - c_2 \overline{D}_{55} \psi_i \psi_j \right) \\ & + (-c_1) \left[\hat{F}_{11} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \hat{F}_{66} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + \hat{F}_{16} \left(\frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) \right] \Bigg\} dxdy \end{aligned}$$

$$\begin{aligned} K_{ij}^{45} = \int_{\Omega^e} \Bigg\{ & \left(\hat{D}_{12} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + \hat{D}_{16} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \hat{D}_{26} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + \hat{D}_{66} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) \\ & + \left(\overline{A}_{45} \psi_i \psi_j - c_2 \overline{D}_{45} \psi_i \psi_j \right) \\ & + (-c_1) \left(\hat{F}_{12} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + \hat{F}_{16} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \hat{F}_{26} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + \hat{F}_{66} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) \Bigg\} dxdy \end{aligned}$$

$$K_{ij}^{51} = \int_{\Omega^e} \left(\hat{B}_{16} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \hat{B}_{66} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + \hat{B}_{12} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} + \hat{B}_{26} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) dxdy$$

$$K_{ij}^{52} = \int_{\Omega^e} \left[\hat{B}_{22} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} + \hat{B}_{66} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \hat{B}_{26} \left(\frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) \right] dxdy$$

$$K_{ij}^{53}=\int_{\Omega^e}\left\{(-c_1)\left[\left(\hat{F}_{12}\frac{\partial\psi_i}{\partial y}\frac{\partial^2\varphi_j}{\partial x^2}+\hat{F}_{22}\frac{\partial\psi_i}{\partial y}\frac{\partial^2\varphi_j}{\partial y^2}+2\hat{F}_{26}\frac{\partial\psi_i}{\partial y}\frac{\partial^2\varphi_j}{\partial x\partial y}\right)\right.\right.$$
$$+\left[\overline{A}_{44}\psi_i\frac{\partial\varphi_j}{\partial y}+\overline{A}_{45}\psi_i\frac{\partial\varphi_j}{\partial x}-c_2\left(\overline{D}_{44}\psi_i\frac{\partial\varphi_j}{\partial y}+\overline{D}_{45}\psi_i\frac{\partial\varphi_j}{\partial x}\right)\right]$$
$$\left.\left.+\left(\hat{F}_{16}\frac{\partial\psi_i}{\partial x}\frac{\partial^2\varphi_j}{\partial x^2}+\hat{F}_{26}\frac{\partial\psi_i}{\partial x}\frac{\partial^2\varphi_j}{\partial y^2}+2\hat{F}_{66}\frac{\partial\psi_i}{\partial x}\frac{\partial^2\varphi_j}{\partial x\partial y}\right)\right]\right\}dxdy$$

$$(K_{NL}^{53})_{ij}=\int_{\Omega^e}\left\{\frac{1}{2}\left[\left(\hat{B}_{12}\frac{\partial w_0}{\partial x}\frac{\partial\psi_i}{\partial y}\frac{\partial\varphi_j}{\partial x}+\hat{B}_{22}\frac{\partial w_0}{\partial y}\frac{\partial\psi_i}{\partial y}\frac{\partial\varphi_j}{\partial y}+\hat{B}_{26}\left(\frac{\partial w_0}{\partial x}\frac{\partial\psi_i}{\partial y}\frac{\partial\varphi_j}{\partial y}+\frac{\partial w_0}{\partial y}\frac{\partial\psi_i}{\partial y}\frac{\partial\varphi_j}{\partial x}\right)\right)\right.\right.$$
$$\left.\left.+\left(\hat{B}_{16}\frac{\partial w_0}{\partial x}\frac{\partial\psi_i}{\partial x}\frac{\partial\varphi_j}{\partial x}+\hat{B}_{26}\frac{\partial w_0}{\partial y}\frac{\partial\psi_i}{\partial x}\frac{\partial\varphi_j}{\partial y}+\hat{B}_{66}\left(\frac{\partial w_0}{\partial x}\frac{\partial\psi_i}{\partial x}\frac{\partial\varphi_j}{\partial y}+\frac{\partial w_0}{\partial y}\frac{\partial\psi_i}{\partial x}\frac{\partial\varphi_j}{\partial x}\right)\right)\right]\right\}dxdy$$

$$K_{ij}^{54}=\int_{\Omega^e}\left\{\left(\hat{D}_{16}\frac{\partial\psi_i}{\partial x}\frac{\partial\psi_j}{\partial x}+\hat{D}_{66}\frac{\partial\psi_i}{\partial x}\frac{\partial\psi_j}{\partial y}+\hat{D}_{12}\frac{\partial\psi_i}{\partial y}\frac{\partial\psi_j}{\partial x}+\hat{D}_{26}\frac{\partial\psi_i}{\partial y}\frac{\partial\psi_j}{\partial y}\right)\right.$$
$$+\left(\overline{A}_{45}\psi_i\psi_j-c_2\overline{D}_{45}\psi_i\psi_j\right)$$
$$\left.+(-c_1)\left(\hat{F}_{16}\frac{\partial\psi_i}{\partial x}\frac{\partial\psi_j}{\partial x}+\hat{F}_{66}\frac{\partial\psi_i}{\partial x}\frac{\partial\psi_j}{\partial y}+\hat{F}_{12}\frac{\partial\psi_i}{\partial y}\frac{\partial\psi_j}{\partial x}+\hat{F}_{26}\frac{\partial\psi_i}{\partial y}\frac{\partial\psi_j}{\partial y}\right)\right\}dxdy$$

$$K_{ij}^{55}=\int_{\Omega^e}\left\{\left[\hat{D}_{22}\frac{\partial\psi_i}{\partial y}\frac{\partial\psi_j}{\partial y}+\hat{D}_{66}\frac{\partial\psi_i}{\partial x}\frac{\partial\psi_j}{\partial x}+\hat{D}_{26}\left(\frac{\partial\psi_i}{\partial x}\frac{\partial\psi_j}{\partial y}+\frac{\partial\psi_i}{\partial y}\frac{\partial\psi_j}{\partial x}\right)\right]\right.$$
$$+\left(\overline{A}_{44}\psi_i\psi_j-c_2\overline{D}_{44}\psi_i\psi_j\right)$$
$$\left.+(-c_1)\left[\hat{F}_{22}\frac{\partial\psi_i}{\partial y}\frac{\partial\psi_j}{\partial y}+\hat{F}_{66}\frac{\partial\psi_i}{\partial x}\frac{\partial\psi_j}{\partial x}+\hat{F}_{26}\left(\frac{\partial\psi_i}{\partial x}\frac{\partial\psi_j}{\partial y}+\frac{\partial\psi_i}{\partial y}\frac{\partial\psi_j}{\partial x}\right)\right]\right\}dxdy$$

(44)

$$M_{ij}^{11}=\int_{\Omega^e}I_0\psi_i\psi_j dxdy\ ,\ M_{ij}^{13}=-c_1\int_{\Omega^e}I_3\psi_i\frac{\partial\varphi_j}{\partial x}dxdy\ ,\ M_{ij}^{14}=\int_{\Omega^e}J_1\psi_i\psi_j dxdy\ ,$$
$$M_{ij}^{22}=\int_{\Omega^e}I_0\psi_i\psi_j dxdy\ ,\ M_{ij}^{23}=-c_1\int_{\Omega^e}I_3\psi_i\frac{\partial\varphi_j}{\partial y}dxdy\ ,\ M_{ij}^{25}=\int_{\Omega^e}J_1\psi_i\psi_j dxdy\ ,$$
$$M_{ij}^{31}=\left(M_{ij}^{13}\right)^T=-c_1\int_{\Omega^e}I_3\frac{\partial\varphi_i}{\partial x}\psi_j dxdy\ ,\ M_{ij}^{32}=\left(M_{ij}^{23}\right)^T=-c_1\int_{\Omega^e}I_3\frac{\partial\varphi_i}{\partial y}\psi_j dxdy\ ,$$
$$M_{ij}^{33}=\int_{\Omega^e}\left[I_0\varphi_i\varphi_j+c_1{}^2I_6\left(\frac{\partial\varphi_i}{\partial x}\frac{\partial\varphi_j}{\partial y}+\frac{\partial\varphi_i}{\partial y}\frac{\partial\varphi_j}{\partial x}\right)\right]dxdy\ ,$$
$$M_{ij}^{34}=-c_1J_4\int_{\Omega^e}\frac{\partial\varphi_i}{\partial x}\psi_j dxdy\ ,\ M_{ij}^{35}=-c_1J_4\int_{\Omega^e}\frac{\partial\varphi_i}{\partial y}\psi_j dxdy\ ,$$
$$M_{ij}^{41}=\left(M_{ij}^{14}\right)^T=\int_{\Omega^e}J_1\psi_i\psi_j dxdy\ ,\ M_{ij}^{43}=\left(M_{ij}^{34}\right)^T=-c_1J_4\int_{\Omega^e}\psi_i\frac{\partial\varphi_j}{\partial x}dxdy\ ,$$
$$M_{ij}^{44}=\int_{\Omega^e}K_2\psi_i\psi_j dxdy\ ,$$

$$M_{ij}^{52} = \left(M_{ij}^{25}\right)^T = \int_{\Omega^e} J_1 \psi_i \psi_j dxdy \ , \ M_{ij}^{53} = \left(M_{ij}^{35}\right)^T = -c_1 J_4 \int_{\Omega^e} \psi_i \frac{\partial \varphi_j}{\partial y} dxdy \ ,$$

$$M_{ij}^{55} = \int_{\Omega^e} K_2 \psi_i \psi_j dxdy \tag{45}$$

$$C_{ij}^{13} = \int_{\Omega^e} \left\{ \left[\sum_{k=1}^{N} \left(Z_{k+1} - Z_k\right) \right] \left[e_m d_m k_c C(t)\right] \frac{\partial \psi_i}{\partial x} \varphi_j \right\} dxdy$$

$$C_{ij}^{23} = \int_{\Omega^e} \left\{ \left[\sum_{k=1}^{N} \left(Z_{k+1} - Z_k\right) \right] \left[e_m d_m k_c C(t)\right] \frac{\partial \psi_i}{\partial y} \varphi_j \right\} dxdy$$

$$C_{ij}^{33} = \int_{\Omega^e} \left\{ c_1 \left[\sum_{k=1}^{N} \frac{1}{4} \left({Z_{k+1}}^4 - {Z_k}^4 \right) \right] \left[e_m d_m k_c C(t)\right] \left(\frac{\partial^2 \varphi_i}{\partial x^2} + \frac{\partial^2 \varphi_i}{\partial y^2} \right) \varphi_j \right\} dxdy$$

$$C_{ij}^{43} = \int_{\Omega^e} \left\{ \left(\left[\sum_{k=1}^{N} \frac{1}{2} \left({Z_{k+1}}^2 - {Z_k}^2 \right) \right] - c_1 \left[\sum_{k=1}^{N} \frac{1}{4} \left({Z_{k+1}}^4 - {Z_k}^4 \right) \right] \right) \left[e_m d_m k_c C(t)\right] \frac{\partial \psi_i}{\partial x} \varphi_j \right\} dxdy$$

$$C_{ij}^{53} = \int_{\Omega^e} \left\{ \left(\left[\sum_{k=1}^{N} \frac{1}{2} \left({Z_{k+1}}^2 - {Z_k}^2 \right) \right] - c_1 \left[\sum_{k=1}^{N} \frac{1}{4} \left({Z_{k+1}}^4 - {Z_k}^4 \right) \right] \right) \left[e_m d_m k_c C(t)\right] \frac{\partial \psi_i}{\partial y} \varphi_j \right\} dxdy \tag{46}$$

$$F_i^1 = \oint_{\Gamma^e} \left(N_{xx} n_x + N_{xy} n_y \right) \psi_i ds \ , \ F_i^2 = \oint_{\Gamma^e} \left(N_{xy} n_x + N_{yy} n_y \right) \psi_i ds \ ,$$

$$F_i^3 = \int_{\Omega^e} q \varphi_i dxdy + \oint_{\Gamma^e} \left(\overline{V}_n \varphi_i + p_{nn} \frac{\partial \varphi_i}{\partial n} \right) ds \ ,$$

$$F_i^4 = \oint_{\Gamma^e} \left(M_{xx} n_x + M_{xy} n_y \right) \psi_i ds \ , \ F_i^5 = \oint_{\Gamma^e} \left(M_{xy} n_x + M_{yy} n_y \right) \psi_i ds$$

$$F_i^{T1} = \int_{\Omega^e} \left(\frac{\partial \psi_i}{\partial x} N_{xx}^T + \frac{\partial \psi_i}{\partial y} N_{xy}^T \right) dxdy \ , \ F_i^{T2} = \int_{\Omega^e} \left(\frac{\partial \psi_i}{\partial x} N_{xy}^T + \frac{\partial \psi_i}{\partial y} N_{yy}^T \right) dxdy \ ,$$

$$F_i^{T4} = \int_{\Omega^e} \left(\frac{\partial \psi_i}{\partial x} M_{xx}^T + \frac{\partial \psi_i}{\partial y} M_{xy}^T \right) dxdy \ , \ F_i^{T5} = \int_{\Omega^e} \left(\frac{\partial \psi_i}{\partial x} M_{xy}^T + \frac{\partial \psi_i}{\partial y} M_{yy}^T \right) dxdy$$

$$F_i^{M1} = \int_{\Omega^e} \left(\frac{\partial \psi_i}{\partial x} N_{xx}^M + \frac{\partial \psi_i}{\partial y} N_{xy}^M \right) dxdy \ , \ F_i^{M2} = \int_{\Omega^e} \left(\frac{\partial \psi_i}{\partial x} N_{xy}^M + \frac{\partial \psi_i}{\partial y} N_{yy}^M \right) dxdy \ ,$$

$$F_i^{M4} = \int_{\Omega^e} \left(\frac{\partial \psi_i}{\partial x} M_{xx}^M + \frac{\partial \psi_i}{\partial y} M_{xy}^M \right) dxdy \ , \ F_i^{M5} = \int_{\Omega^e} \left(\frac{\partial \psi_i}{\partial x} M_{xy}^M + \frac{\partial \psi_i}{\partial y} M_{yy}^M \right) dxdy \tag{47}$$

This completes the nonlinear finite element model development of the third-order shear deformation plate theory. As in the case of linear finite element analysis, the nonlinear transient equations may be solved using the Newmark time integration schemes.

7 CONCLUSIONS

A unified third-order plate theory is used to develop a displacement finite element model of laminated composite plates with smart material layers for transient response. The unified formulation includes the classical, first-order, and third-order shear deformation plate theories as special cases. The smart material used in this study to achieve damping of transverse deflection is the Terfenol-D magnetostrictive material, although in principle any other actuating material can be used. The negative velocity feedback control is used to damp the transient response. A number of parametric studies were carried out to understand the damping characteristics of various laminates with embedded smart-material layers. Some of the observations are summarized below.

Use of quarter plate models in place of full plate models was studied first. It is found that for antisymmetric cross-ply, angle-ply, and general angle-ply laminates and symmetric cross-ply laminates with simply supported boundary conditions, a quadrant of the plate with proper symmetry boundary conditions may be used to reduce the computational effort.

The maximum damping of deflection occurs when the smart material layers are placed farthest from the midplane because the bending moments generated by the smart material layer are maximum when they are away from the midplane. It is observed that for a lower value of the feedback coefficient the time taken to damp deflection is longer. The damping tendencies of symmetric cross-ply, angle-ply and general angel-ply composite laminates have been found to be similar. The damping characteristics of a fully clamped composite plate are similar to that of a simply supported composite plate.

REFERENCES

1. Maugin, G. A., 1988, *Continuum Mechanics of Electro-magnetic Solids*, North-Holland, Amesterdam.
2. Uchino, K., 1986, "Electrostrictive Actuators: Materials and Applications," *Cer. Bull.*, Vol. 65, pp. 647-652.
3. Goodfriend, M. J., and Shoop, K. M., 1992, "Adaptive characteristics of the Magnetostrictive Alloy, Terfenol-D, for Active Vibration Control," *Journal of Intelligent Material Systems and Structures*, Vol. 3, pp. 245-254.
4. Anjanappa, M., and Bi, J., 1994, "A Theoretical and Experimental Study of Magnetostrictive Mini Actuators," *Smart Materials and Structures*, Vol. 1, pp. 83-91.
5. Anjanappa, M., and Bi, J., 1994, "Magnetostrictive Mini Actuators for Smart Structural Application," *Smart Materials and Structures*, Vol. 3, pp. 383-390.
6. Pratt, J. R., and Flatau, A. B., 1995, "Development and Analysis of Self-Sensing magnetostrictive Actuator Design," *Journal of Intelligent Material Systems and Structures*, Vol. 6, pp. 639-648.
7. Eda, H., Kobayashi, T., Nakamura, H., and Akiyama, T., 1995, "Giant Magnetostriction Compounds with Structure Textured by Resin Bound on Giant Magnetostriction Fine Powder in Magnetic Field and its Actuator," *Transactions of Japanese Society for Mechanical Engineering, Series C*, Vol. 61, pp. 168-17.
8. Krishna Murty, A.V., Anjanappa, M., and Wu, Y. -F., 1997, "The Use of Magnetostrictive Particle Actuators for Vibration Attenuation of Flexible Beams," *Journal of Sound and Vibration*, Vol. 206, pp. 133-149.

9. Krishna Murty, A. V., Anjanappa, M., Wu, Y. -F., 1998, Bhattacharya, B., and Bhat, M.S., "Vibration Suppression of Laminated Composite Beams Using Embedded Magneto-strictive Layers," *Journal A-S* , Vol. 78, pp. 38-44.
10. Reddy, J. N. and Barbosa, J. A., 2000,``Vibration Suppression of Laminated Composite Beams," *Smart Materials and Structures*, Vol. 9, pp. 49-58.
11. Reddy, J.N., *Energy Principles and Variational Methods in Applied Mechanics*, Wiley, New York, 2002.
12. Reddy, J.N., *Mechanics of Laminated Composite Plates: Theory and Analysis*, CRC Press, Boca Raton, Florida, 1997.
13. Reddy, J.N., ``On Laminated Composite Plates with Integrated Sensors and Actuators," *Engineering Structures*, Vol. 21, 1999, pp. 568-593.
14. Rostam-Abadi, F., Reddy, J.N. and Lee, S.J., ``Vibration Suppression of Cross-Ply Laminated Plates with Magnetostrictive Layers," *Proceedings of SECTAM* XXI, May 2002.